Study of Coulomb collisions and magneto-ionic propagation effects on ISR measurements at Jicamarca

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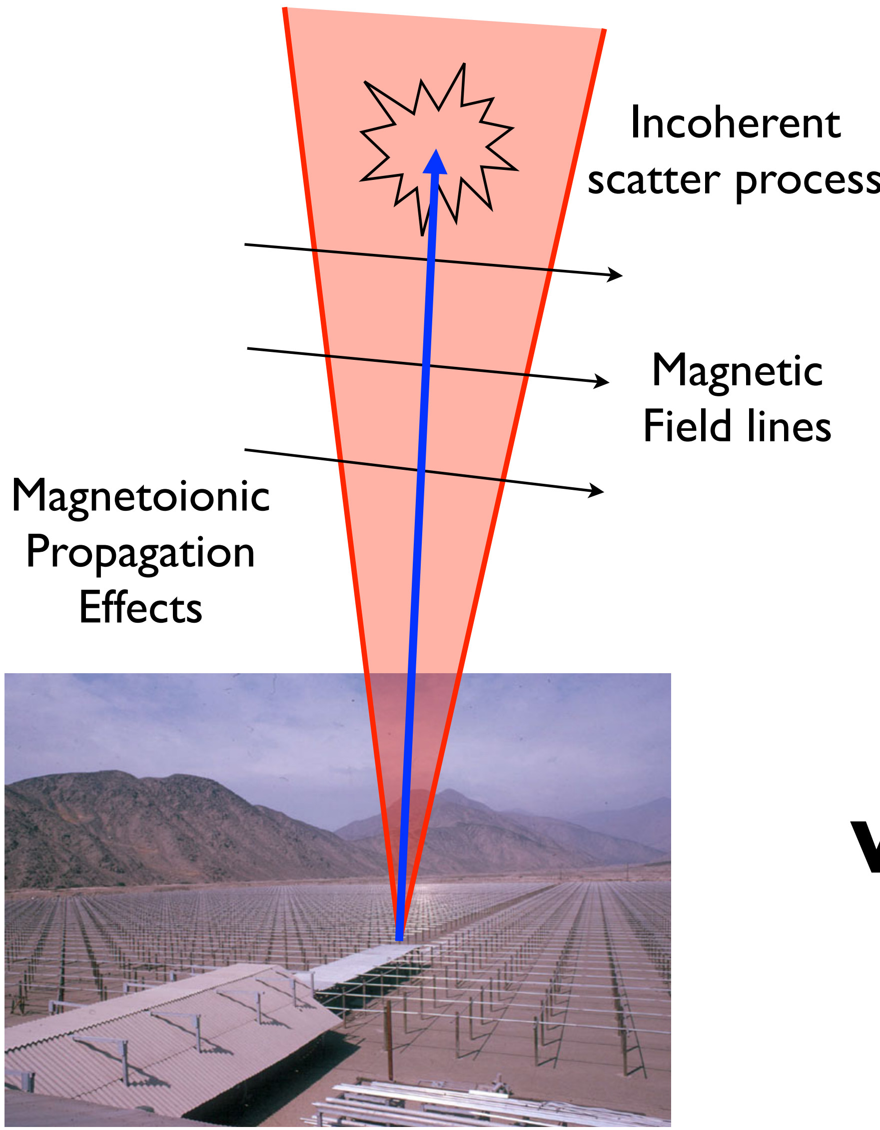
Jicamarca ISR measurements perp. to B

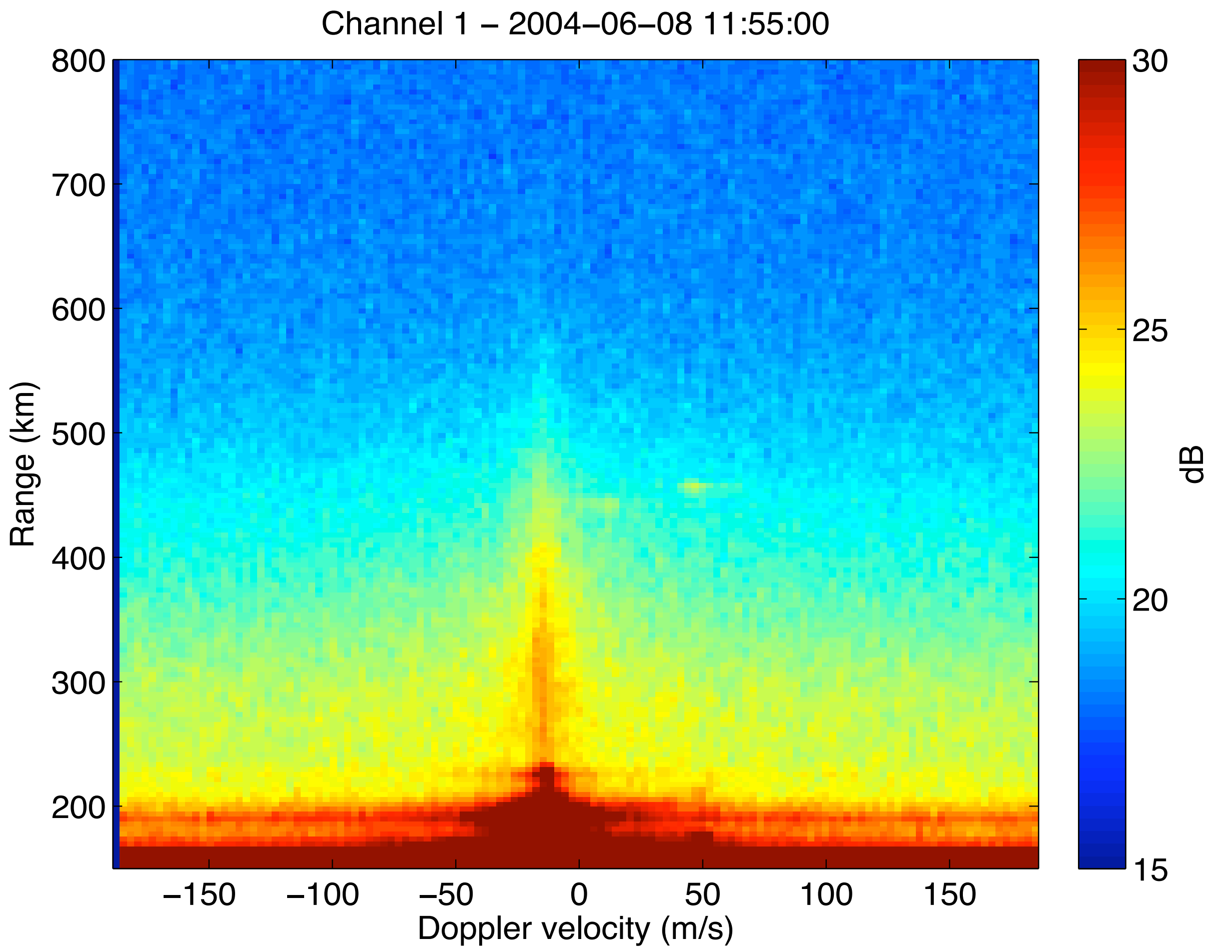


Physical parameters that can be measured:

- Drifts: using Kudeki et al (1999) spectral fitting technique (Doppler shift of ISR spectrum).
- Densities: using the "Differential-phase" technique developed by Kudeki et al (2003).

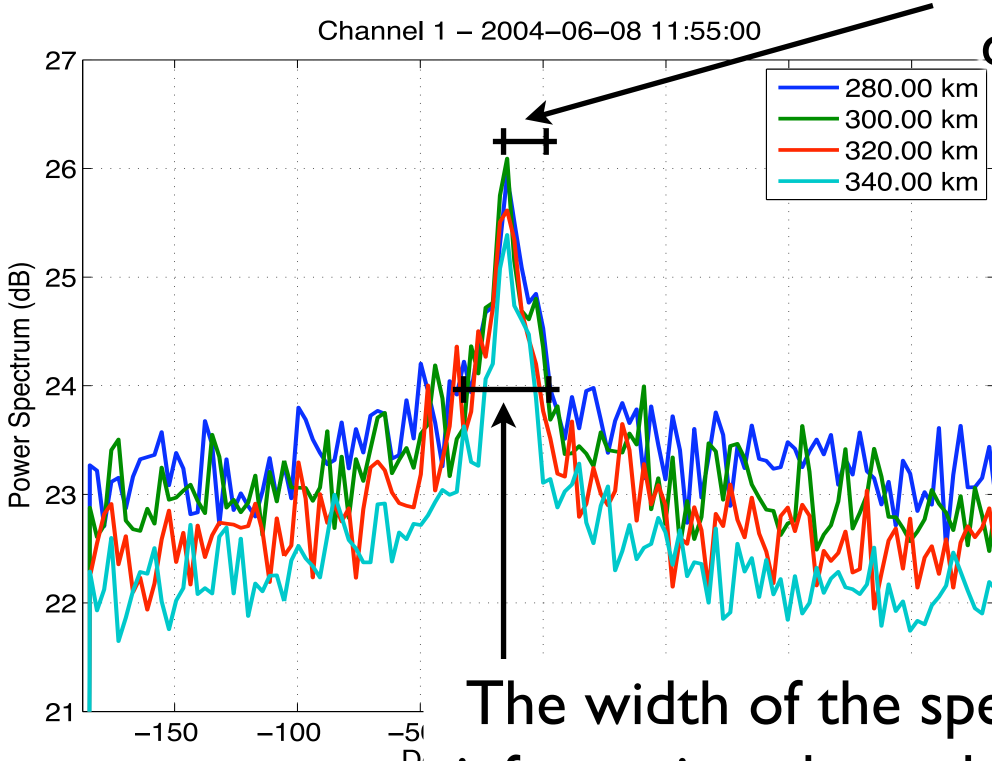
What about temperatures?

Jicamarca ISR spectrum perp. to B



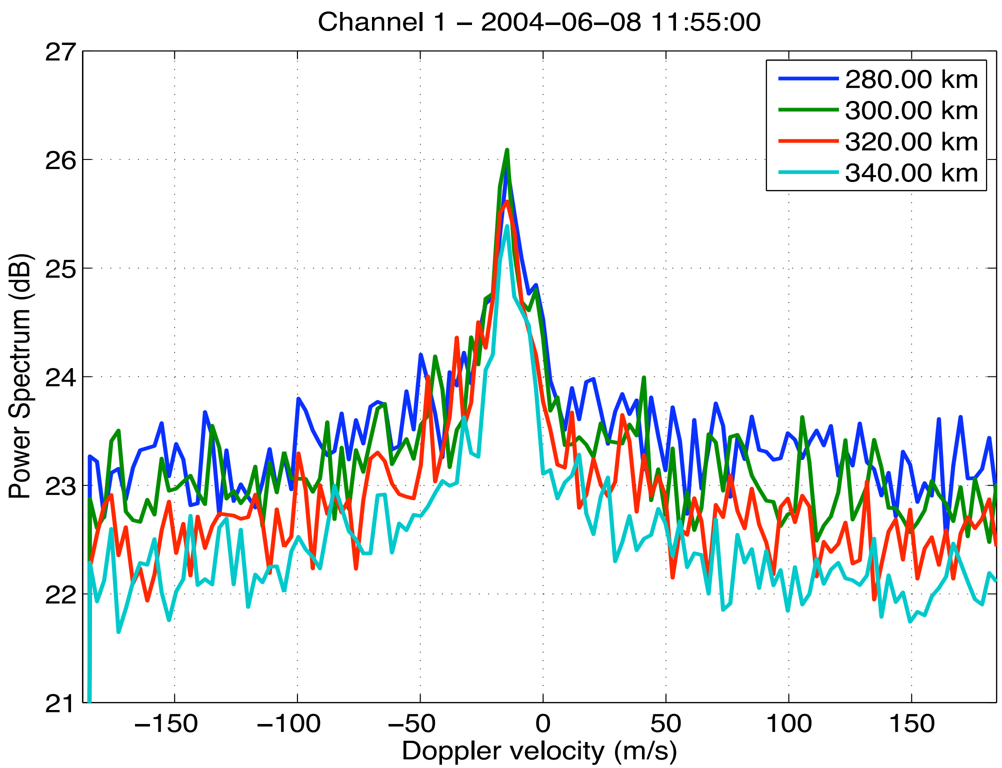
Jicamarca ISR spectrum perp. to B

Doppler shift of the spectrum is a direct measurement of the drift.



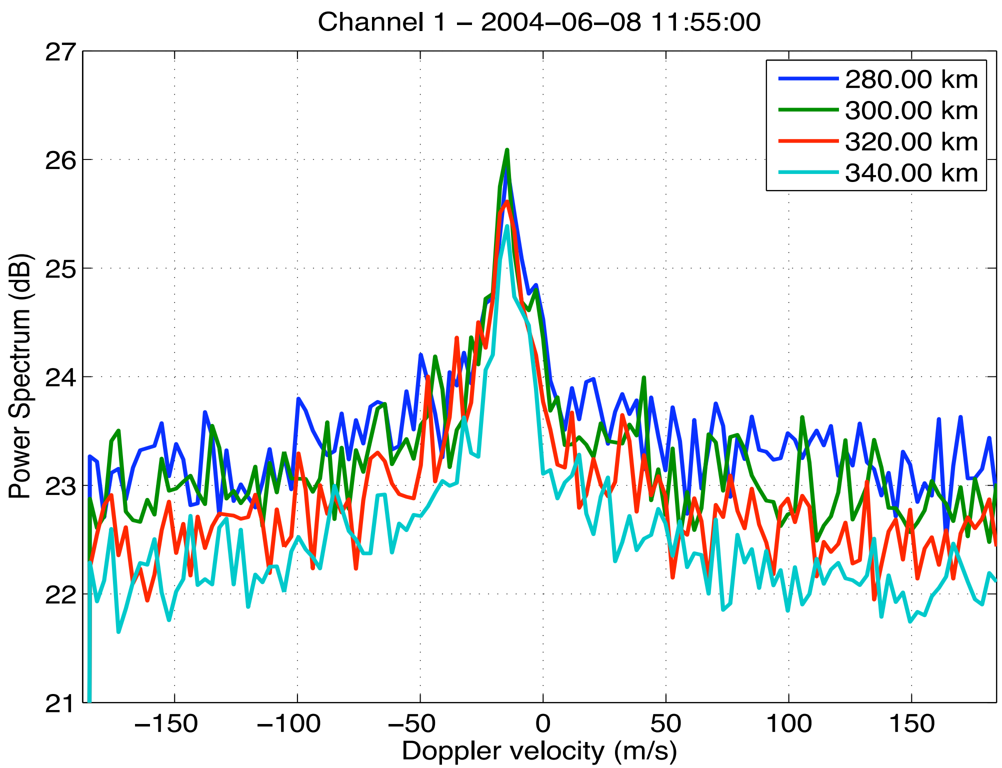
The width of the spectrum contains information about the temperatures.

Jicamarca ISR spectrum perp. to B



Kudeki et al (1999) fitted the measurements using a simplified spectral model. This model was developed based on the collisionless IS theory. But, the temperatures they obtained were about half of what is expected.

Jicamarca ISR spectrum perp. to B



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The measured spectrum was narrower than what the collisionless theory predicts, and therefore, a revision of the IS theory for modes propagating perpendicular to B was needed.

Research project

- Main goal: To develop an incoherent scatter spectrum model for modes propagating perpendicular to B.

- This project is divided in two stages.
  - Study of the effects of Coulomb collisions
  - Modeling the magnetoionic propagation effects

- Future Application: The estimation of ionospheric temperatures with Jicamarca antenna beams pointed perpendicular to B.

First stage:

Modeling the incoherent scatter spectrum considering the effects of Coulomb collisions

IS spectrum and Gordeyev integrals

- Spectrum of electron density fluctuations (e.g., Kudeki & Milla, 2010)

$$\langle |n_e(\vec{k},\omega)|^2\rangle = \frac{|j\omega\epsilon_o + \sigma_i(\vec{k},\omega)|^2\langle |n_{te}(\vec{k},\omega)|^2\rangle + |\sigma_e(\vec{k},\omega)|^2\langle |n_{ti}(\vec{k},\omega)|^2\rangle}{|j\omega\epsilon_o + \sigma_e(\vec{k},\omega) + \sigma_i(\vec{k},\omega)|^2}$$

- In terms of the Gordeyev integrals, the spectra of thermal density fluctuations and the conductivities are given by

$$\frac{\langle |n_{ts}(\vec{k},\omega)|^2\rangle}{N_s} = 2\mathrm{Re}\{J_s(\omega)\} \qquad \frac{\sigma_s(\omega,\vec{k})}{j\omega\epsilon_o} = \frac{1 - j\omega J_s(\omega)}{k^2 h_s^2}$$

- The Gordeyev integral is the one-sided Fourier transform of the single particle ACF (Hagfors & Brockelman, 1971)

$$J_s(\omega) = \int_0^\infty d\tau e^{-j\omega\tau}\langle e^{j\vec{k}\cdot\Delta\vec{r}_s}\rangle \qquad \langle e^{j\vec{k}\cdot\Delta\vec{r}_s}\rangle = \langle e^{j\vec{k}\cdot(\vec{r}_s(t+\tau)-\vec{r}_s(t))}\rangle$$

Langevin equation and particle trajectories

- Instead of solving the Boltzmann kinetic equation with the Fokker-Planck collision operator to determine the pdf of the particle displacements, we model the particle motion by a set of Generalized Langevin equations:

$$\begin{aligned}\frac{d\vec{v}(t)}{dt} &= \frac{q}{m}\,\vec{v}(t)\times\vec{B} - \beta(v)\,\vec{v}(t) + \sqrt{D_{\|}(v)}\,\mathcal{W}_1(t)\,\hat{v}_{\|}(t) \\ &\quad + \sqrt{\frac{D_{\perp}(v)}{2}}\,\mathcal{W}_2(t)\,\hat{v}_{\perp 1}(t) + \sqrt{\frac{D_{\perp}(v)}{2}}\,\mathcal{W}_3(t)\,\hat{v}_{\perp 2}(t) \\ \frac{d\vec{r}(t)}{dt} &= \vec{v}(t)\end{aligned}$$

- Coulomb collisions are simulated by a deterministic friction force and random diffusion forces.
- These equations represent an alternative description of the Fokker-Planck collision process (Chandrasekhar, 1943; Gillespie, 1996).
- This approach give us more insight into the physics of the problem.

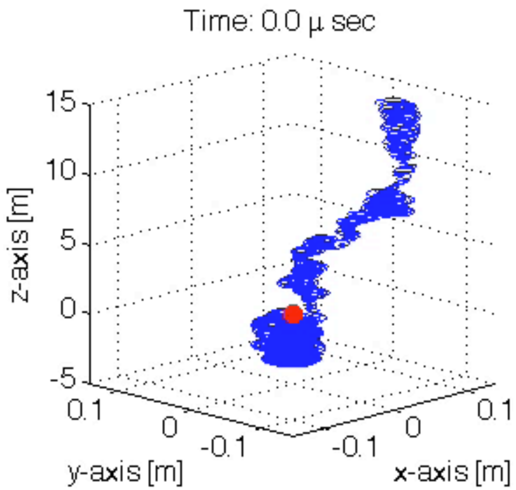
Computer simulations

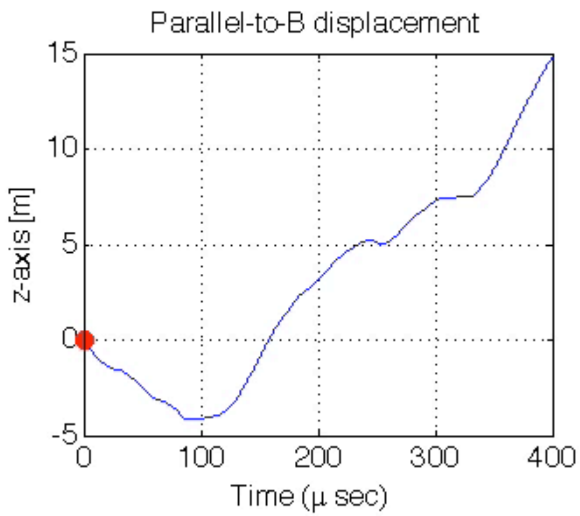
- The trajectory of a test particle is simulated using

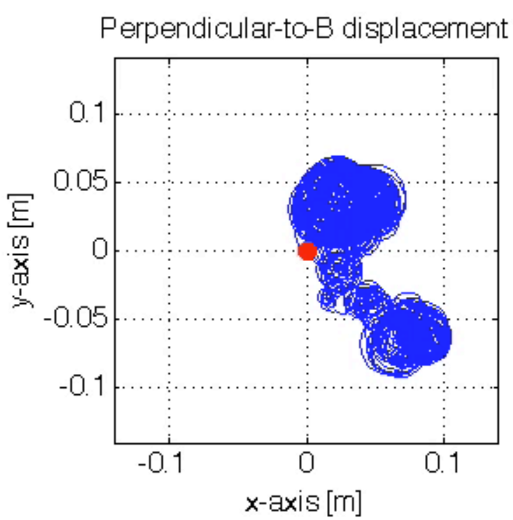
$$\begin{aligned}\vec{v}_{n+1} &= \vec{v}_n + \frac{q}{m}\,\vec{v}_n \times \vec{B}\,\Delta t - \beta(v_n)\,\Delta t\,\vec{v}_n + \sqrt{D_{\|}(v_n)\,\Delta t}\,\mathcal{N}_1\,\hat{v}_{\|} + \\ &\quad \sqrt{D_{\perp}(v_n)\,\frac{\Delta t}{2}}\,\mathcal{N}_2\,\hat{v}_{\perp 1} + \sqrt{D_{\perp}(v_n)\,\frac{\Delta t}{2}}\,\mathcal{N}_3\,\hat{v}_{\perp 2} \\ \vec{r}_{n+1} &= \vec{r}_n + \frac{\vec{v}_{n+1} + \vec{v}_n}{2}\,\Delta t\end{aligned}$$

- The Spitzer velocity-dependent friction and diffusion coefficients are used to model the effects of Coulomb collisions.
- Assumption: the magnetic field is weak enough such that within a Debye cube the trajectories of electrons and ions exhibit small curvatures due to the magnetic field.
- For a given plasma configuration, the simulations run for several hours in order to obtain good statistics of the particle trajectories.

3-D particle trajectories







10^4 sequences of 2^{17} samples are generated (30 GB), however, only the statistics (ACF's) are stored (60 MB).

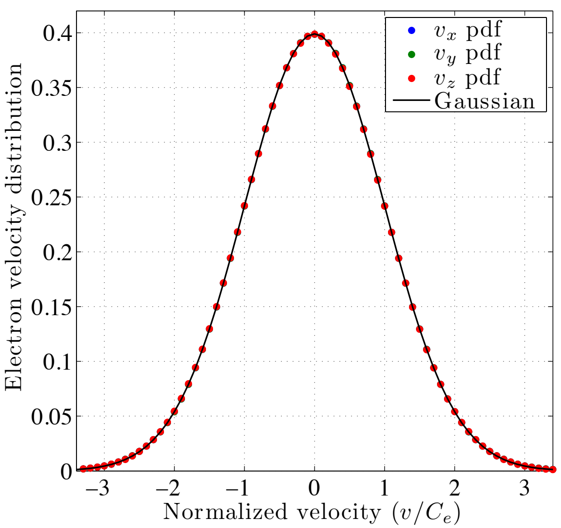
O+ Plasma:

$N_e = 10^{12}$ m^{-3}

T_e = 1000 K

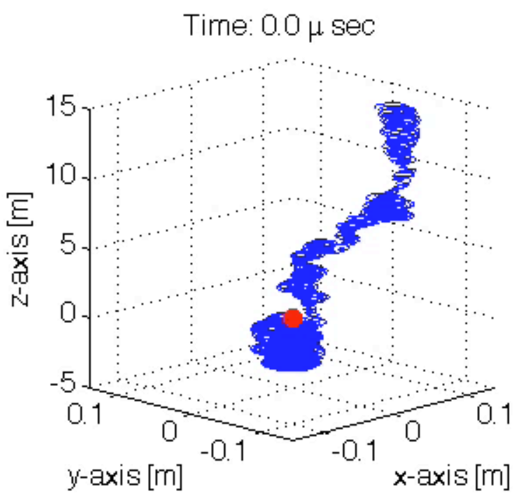
T_i = 1000 K

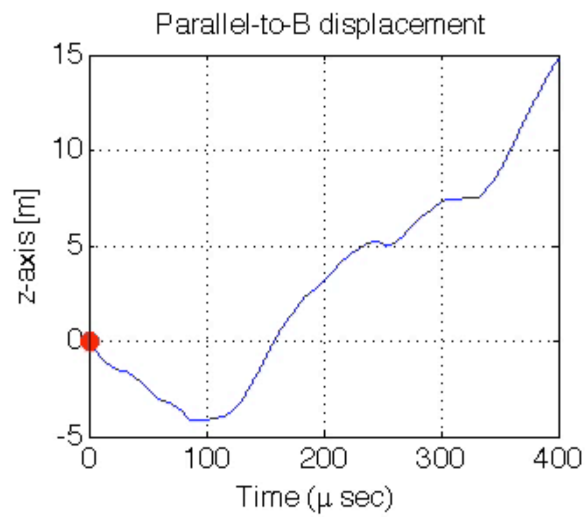
B = 25 000 nT



Velocity distributions have a gaussian shape.

3-D particle trajectories





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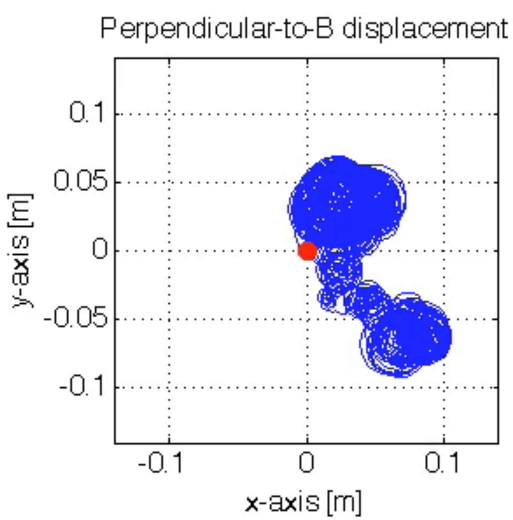

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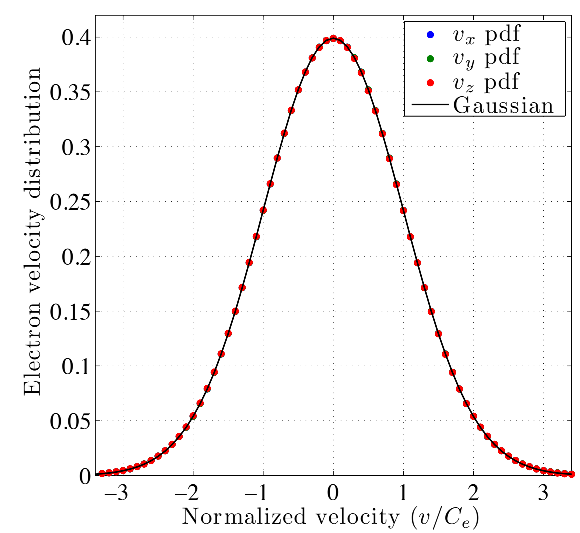
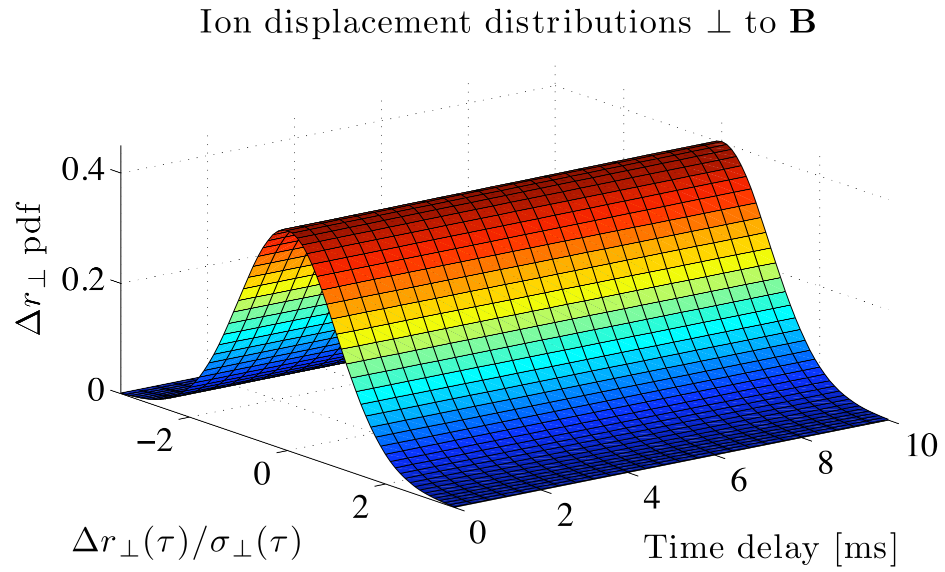


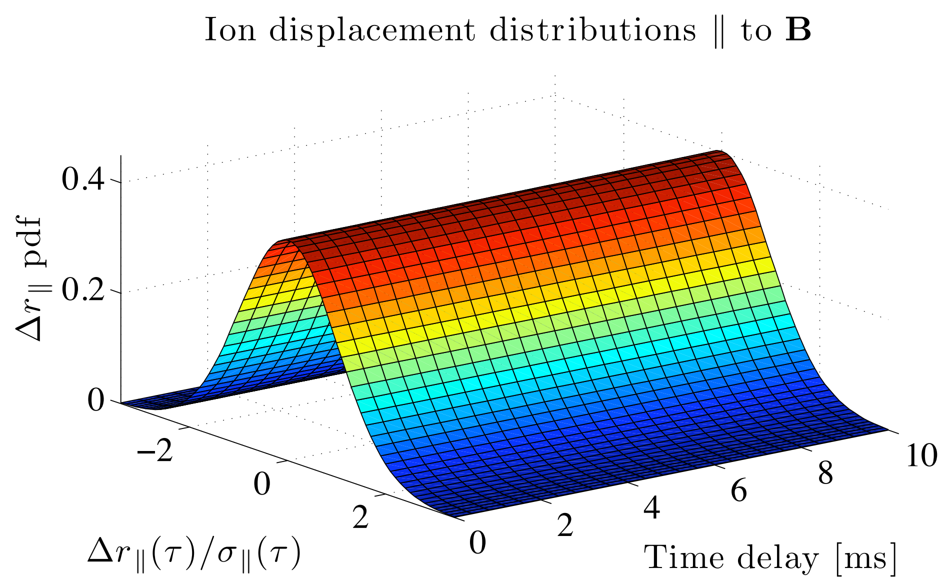
Velocity distributions have a gaussian shape.

Statistics of ion trajectories

- The pdf of the displacement in the direction perpendicular to B is gaussian as a function of delay τ.
- In the parallel direction, the pdf also looks gaussian.
- A Brownian motion model with Gaussian trajectories is a good representation of the ion motion process (Woodman, 1967).
- The single-ion ACF can be approximated by

$$\left\langle e^{j\vec{k}\cdot\Delta\vec{r}} \right\rangle = e^{-\frac{1}{2}k^2 \sin^2\alpha \langle \Delta r_\parallel^2 \rangle} \times e^{-\frac{1}{2}k^2 \cos^2\alpha \langle \Delta r_\perp^2 \rangle}$$



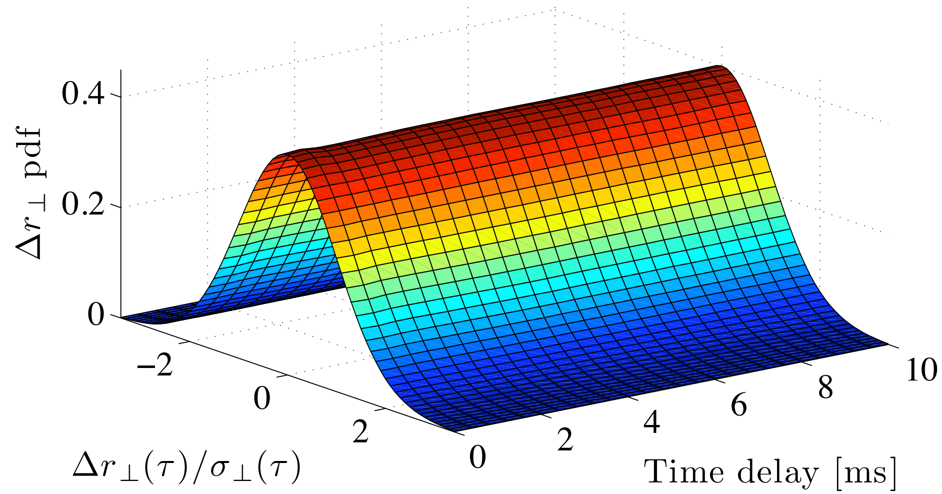


Statistics of electron trajectories

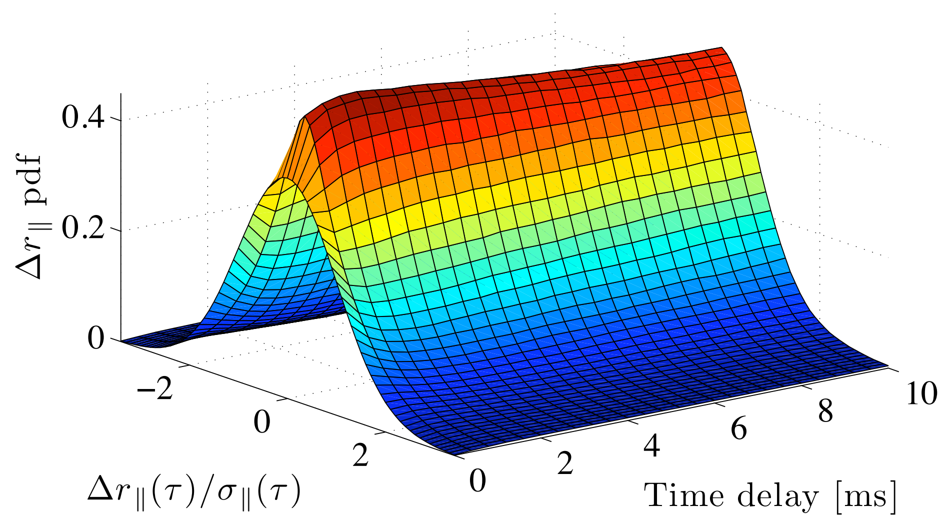
- The pdf of the displacement in the direction perp. to B is almost gaussian as function of delay τ.
- In the parallel direction, the pdf looks gaussian at short τ, but becomes narrow in less than 1 ms.
- Brownian motion is not a good model for the electrons.
- The components of the electron vector displacement are not independent variables.

$$\xcancel{\left\langle e^{j\vec{k}\cdot\Delta\vec{r}} \right\rangle = e^{-\frac{1}{2}k^2 \sin^2\alpha \langle \Delta r_{\parallel}^2 \rangle} \times e^{-\frac{1}{2}k^2 \cos^2\alpha \langle \Delta r_{\perp}^2 \rangle}}$$

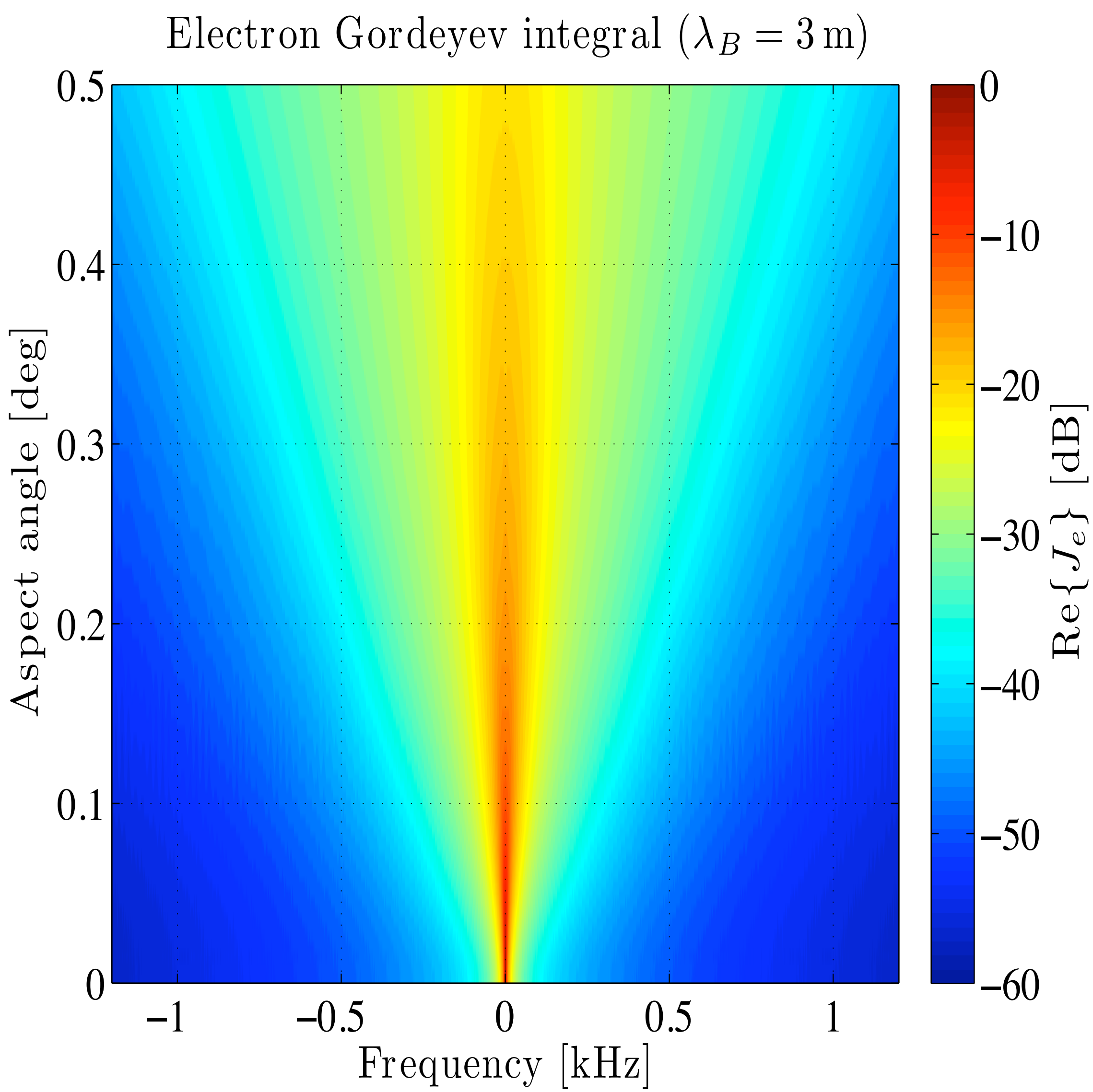
Electron displacement distributions $\perp$ to $\mathbf{B}$



Electron displacement distributions $\parallel$ to $\mathbf{B}$

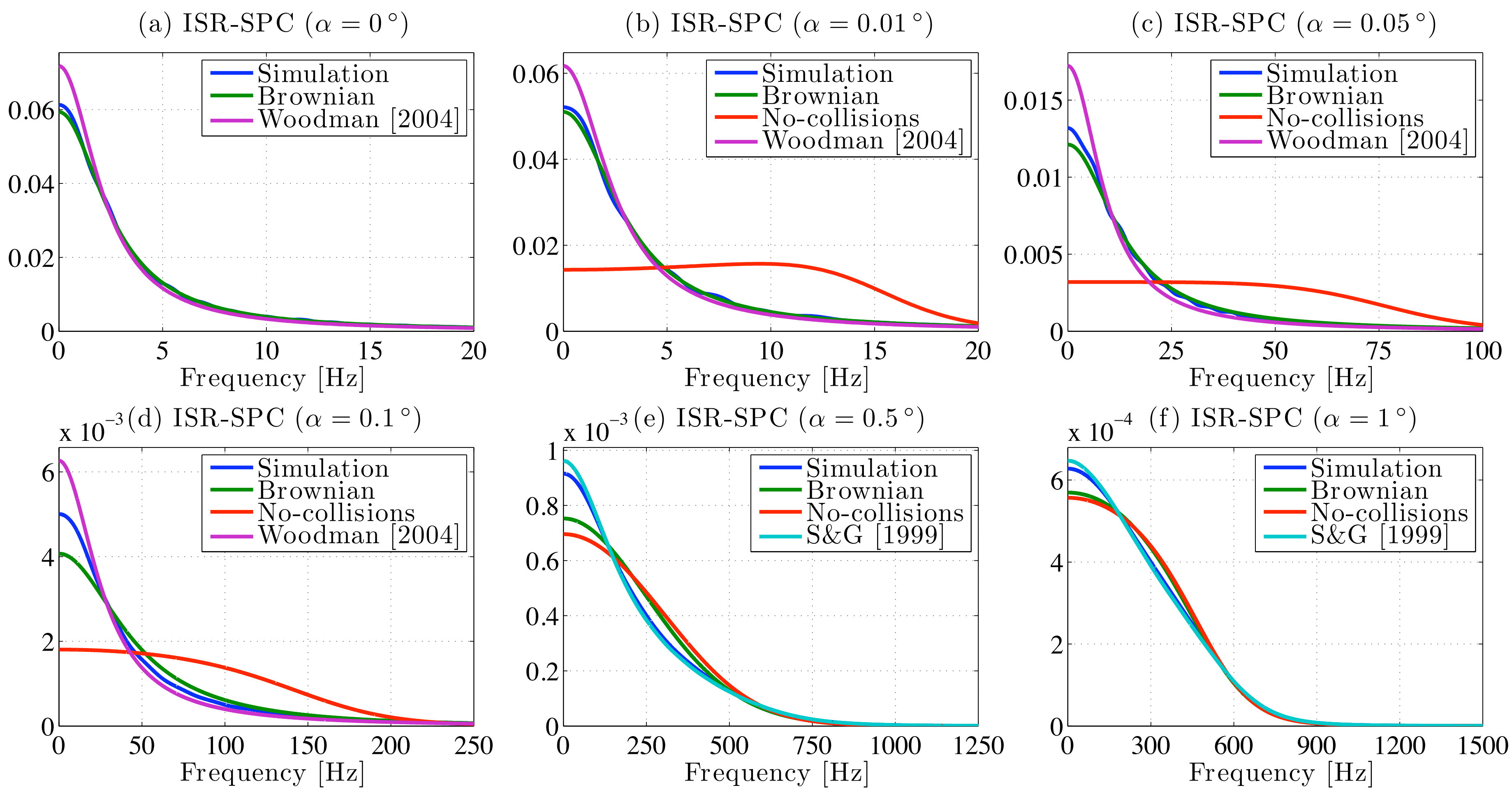


Database of single-electron ACF's



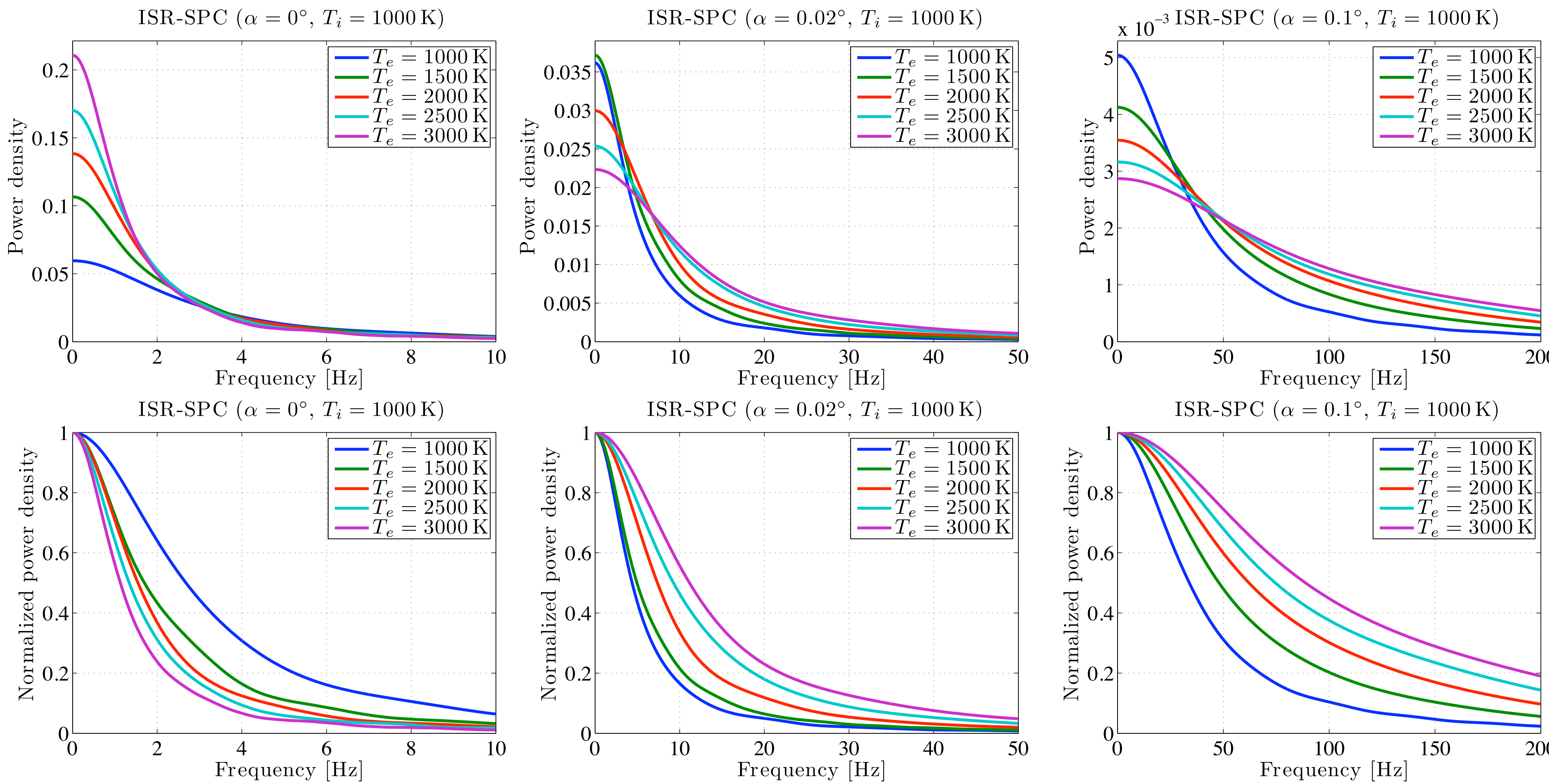
- We have built a library for an oxygen plasma that considers
  - $11 < \log_{10}(Ne) < 13$
  - 600K < Te < 3000K
  - 600K < Ti < 2000K
  - |B| = 20, 25, 30 μT
  - Large set of aspect angles from 0° to 90°.
- Electron Gordeyev integrals are computed using the Chirp-Z transform (Li et al, 1991)
- A web-page with the results http://collisions.csl.uiuc.edu/database/gordeyev/

IS collisional spectra (1)



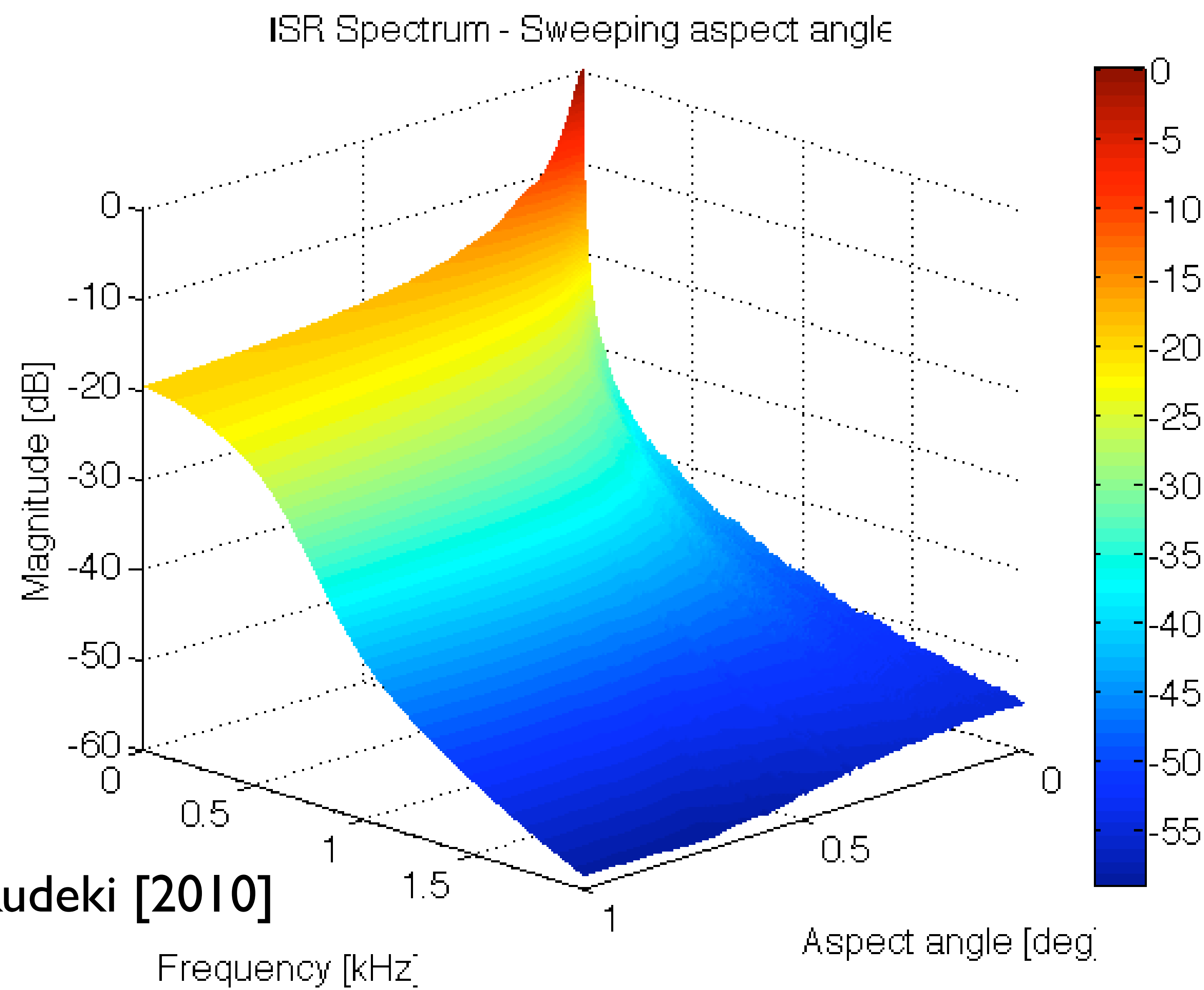
Simulated spectrum at different magnetic aspect angles: (a) α=0°, (b) α=0.01°, (c) α=0.05°, (d) α=0.1°, (e) α=0.5°, and (f) α=1° (Milla & Kudeki, 2010).

IS collisional spectra (2)



Electron temperature dependence of the simulated IS spectrum for λ_B=3m at aspect angles α=0° (left panels), α=0.02° (central panels), and α=0.1° (right panels) (Milla & Kudeki, 2010).

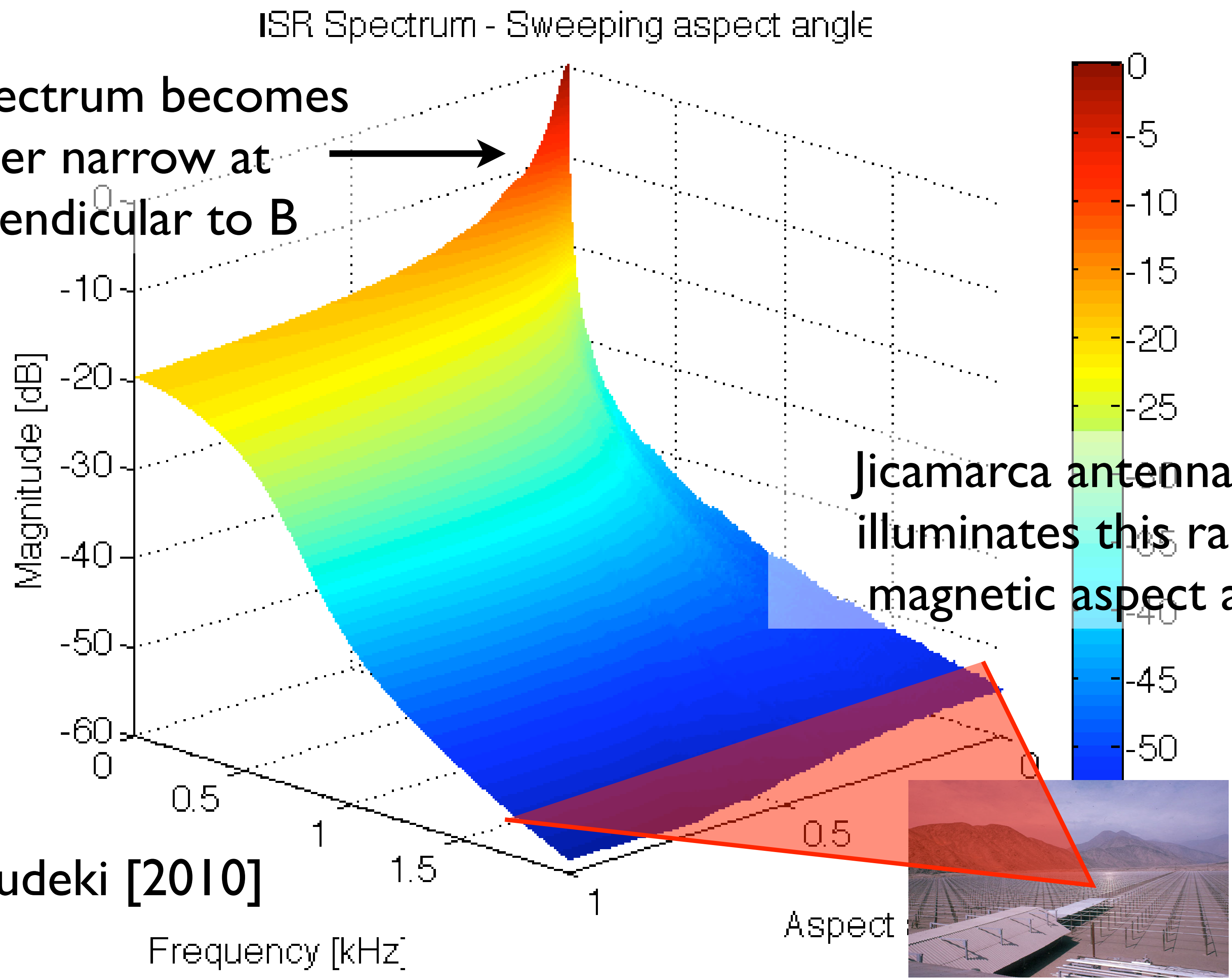
Collisional IS Spectrum



Milla & Kudeki [2010]

Collisional IS Spectrum

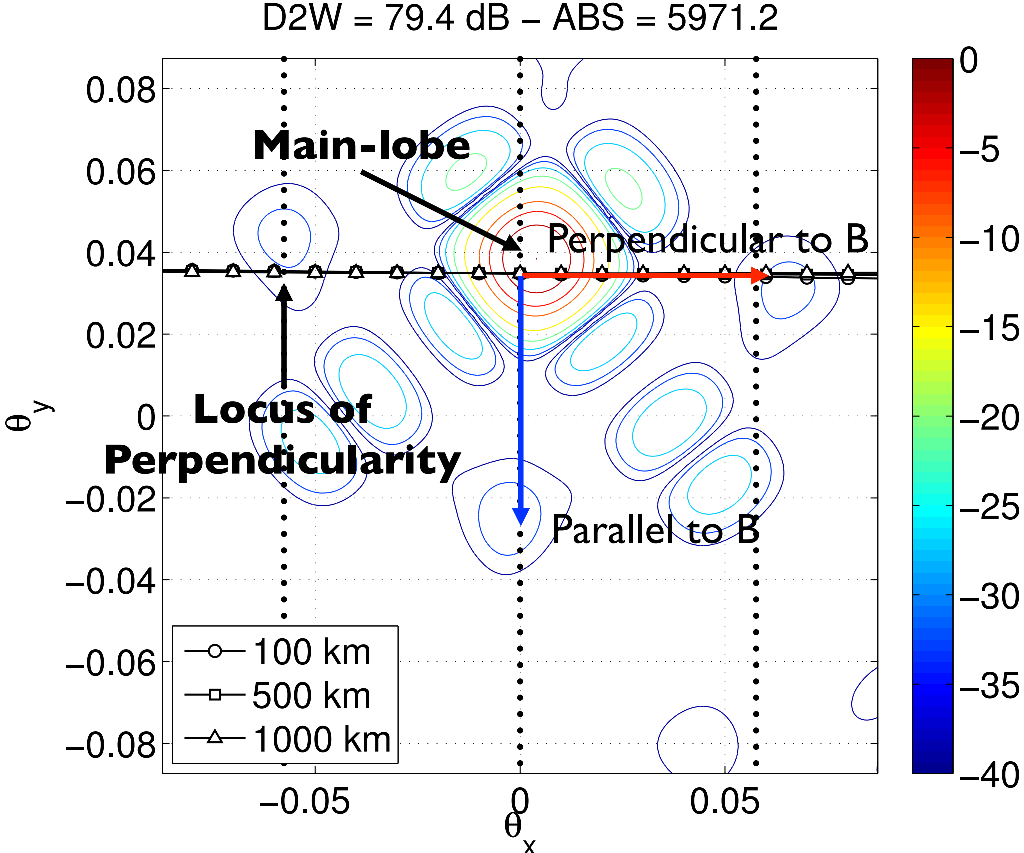
The spectrum becomes super narrow at perpendicular to B



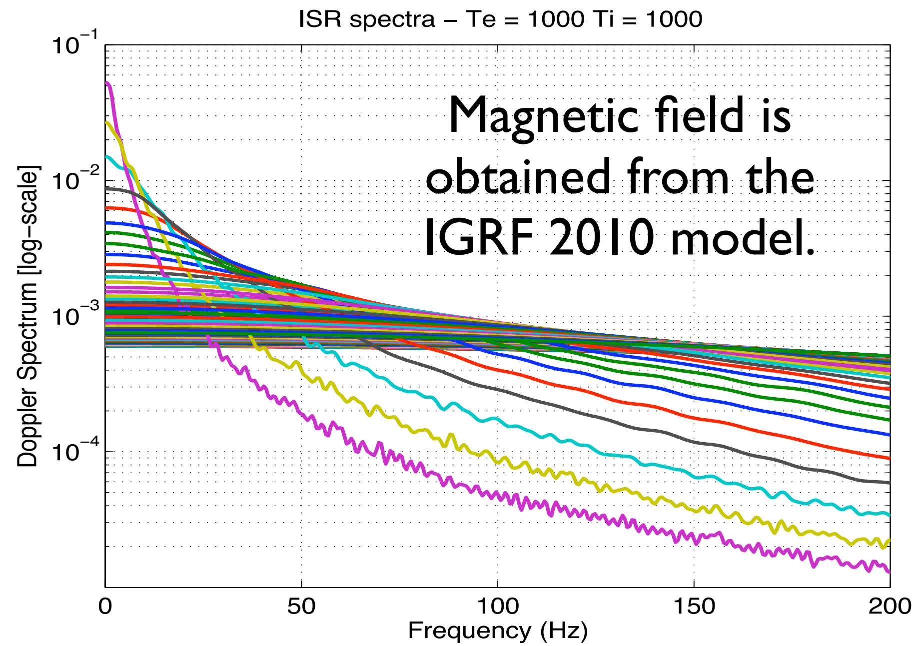
Jicamarca antenna beam illuminates this range of magnetic aspect angles

Milla & Kudeki [2010]

Beam-weighted ISR spectrum



- Jicamarca antenna beam pointed perp to B (beam-width: ~1 deg).
- The measured spectrum is the sum of the spectra corresponding to different magnetic aspect angles.

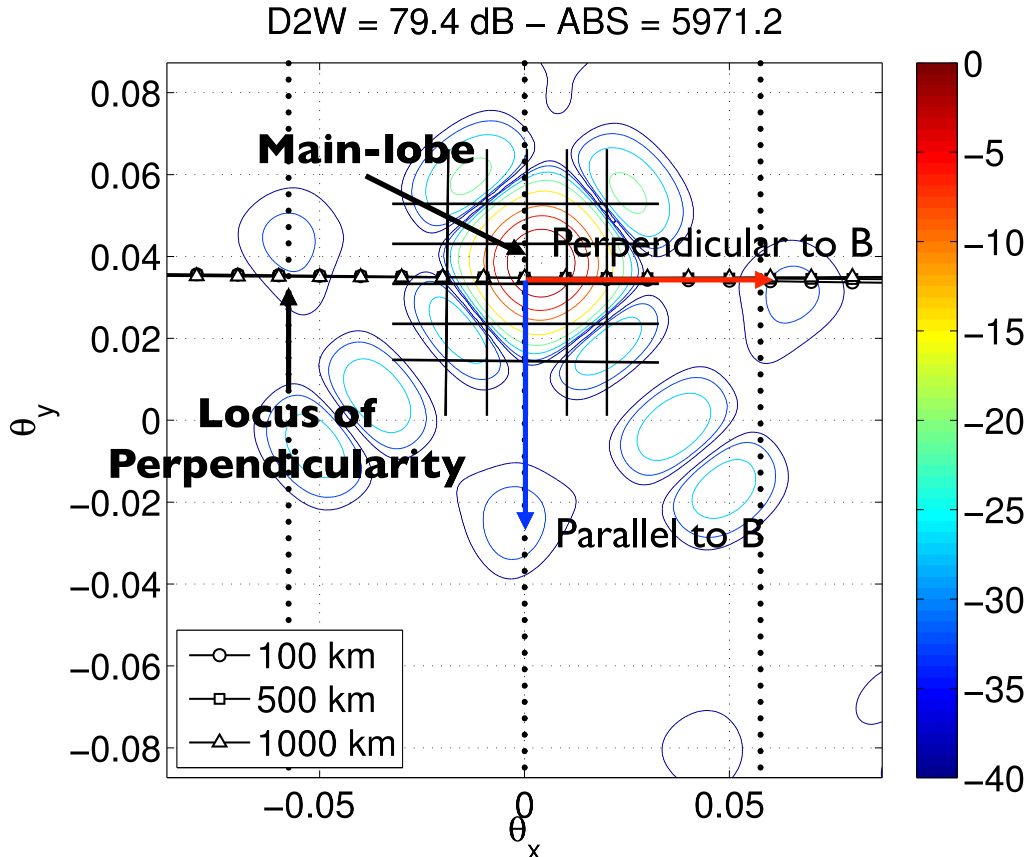


Radar equation:

$$\frac{S(\omega)}{E_t K} = \frac{\delta R}{R^2} \int d\Omega\, W(\hat{\mathbf{r}})\, \sigma(\vec{k}, \omega)$$

IS-RCS → $\sigma(\vec{k}, \omega) = 4\pi r_e^2 \langle |n_e(\vec{k}, \omega)|^2 \rangle$

Beam-weighted ISR spectrum



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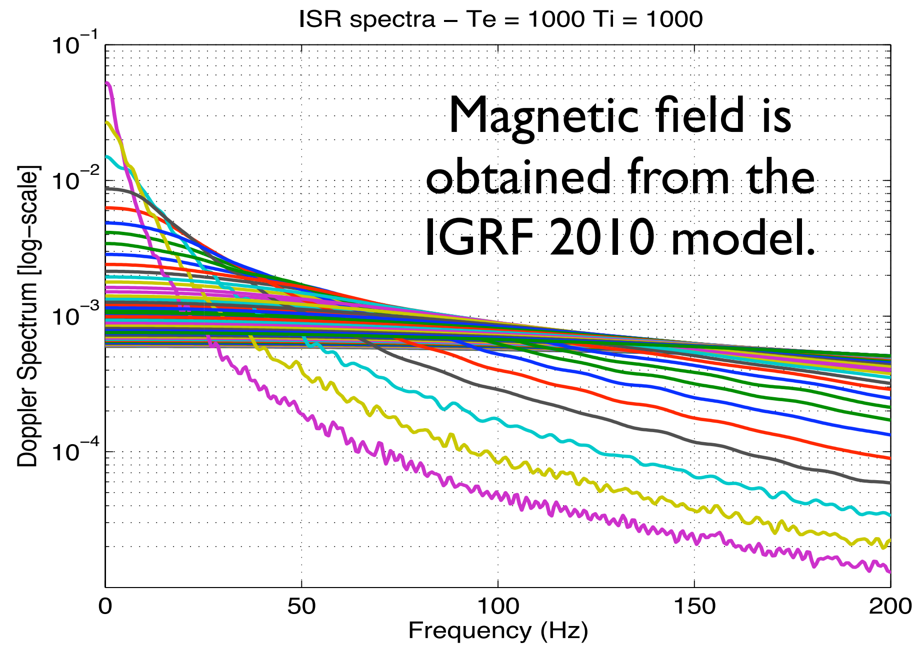

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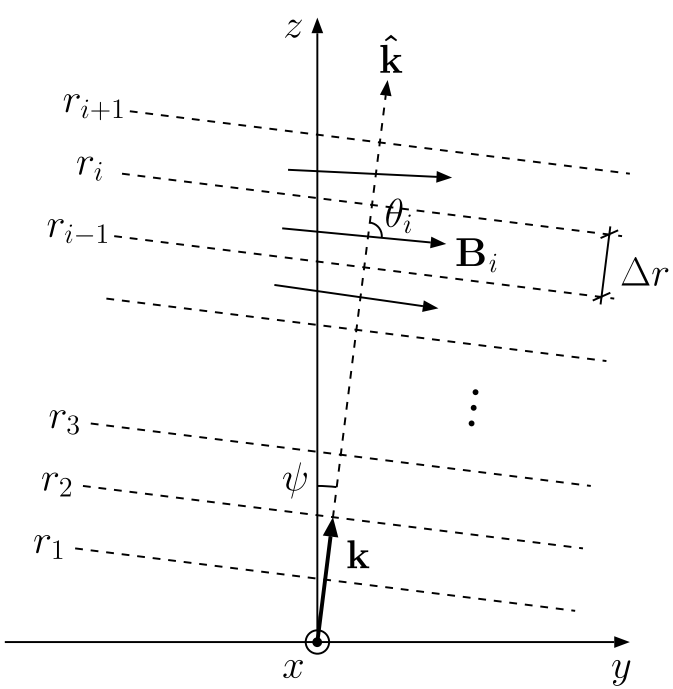
Second stage:

Modeling the magneto-ionic propagation effects on the beam-weighted incoherent scatter radar spectrum

Magneto-ionic propagation model (I)



Geometry of wave propagation in an inhomogeneous magnetized ionosphere.

Appleton-Hartree Solution

$$Y_L = Y\cos\theta, \quad Y_T = Y\sin\theta, \quad Y = \frac{\Omega}{\omega}, \quad X = \frac{\omega_p^2}{\omega^2}$$

$$F_O = F_1 - F_2, \quad F_X = F_1 + F_2, \quad F_1 = \frac{Y_T^2/2}{1-X}, \quad F_2^2 = F_1^2 + Y_L^2$$

$$n_{O,X}^2 = 1 - \frac{X}{1 - F_{O,X}}$$

$$\Delta n = \frac{n_O - n_X}{2} \qquad \bar{n} = \frac{n_O + n_X}{2} \qquad a = \frac{F_O}{Y_L}$$

$$\begin{bmatrix} E_\theta^i \\ E_\phi^i \end{bmatrix} = \underbrace{\frac{e^{-jk_o\bar{n}r}}{1+a^2}\begin{bmatrix} e^{-jk_o\Delta nr} + a^2 e^{jk_o\Delta nr} & 2a\sin(k_o\Delta nr) \\ -2a\sin(k_o\Delta nr) & a^2 e^{-jk_o\Delta nr} + e^{jk_o\Delta nr} \end{bmatrix}}_{\bar{\mathbf{T}}_i} \begin{bmatrix} E_\theta^{i-1} \\ E_\phi^{i-1} \end{bmatrix}$$

Backscattered electric field for every propagation direction $\longrightarrow$ $\vec{E}_o^r \propto \kappa_i \underbrace{\bar{\mathbf{T}}_1\bar{\mathbf{T}}_2\cdots\bar{\mathbf{T}}_i\bar{\mathbf{T}}_i\cdots\bar{\mathbf{T}}_2\bar{\mathbf{T}}_1}_{\bar{\mathbf{\Pi}}_i} \vec{E}_o^t$

Two-way propagator matrix $\rightarrow$ $\bar{\mathbf{\Pi}}_i$

Magneto-ionic propagation model (2)

Soft-Target Radar equation:

$$\frac{S(\omega)}{E_t K} = \frac{\delta R}{R^2} \int d\Omega\, W(\vec{r})\, \sigma(\vec{k},\omega) \qquad \sigma(\vec{k},\omega) = 4\pi r_e^2 \left\langle |n_e(\vec{k},\omega)|^2 \right\rangle$$

But now, $W(\vec{r})$ is an effective two-way radiation pattern

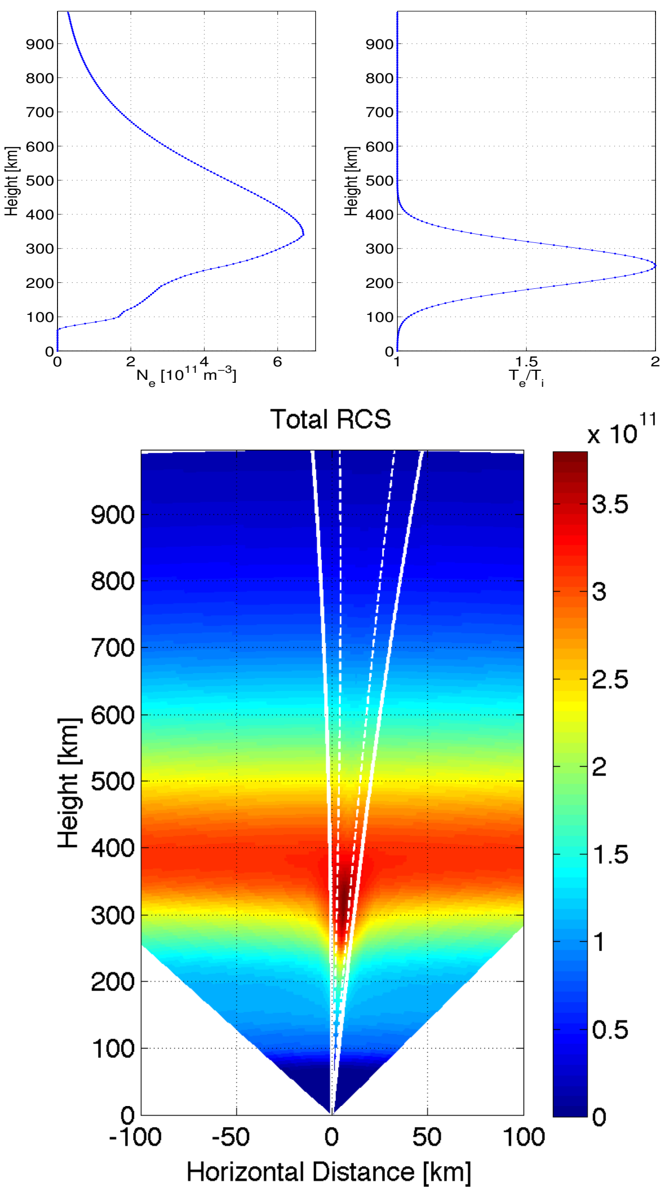
$$W(\vec{r}) = \frac{1}{k^2} G_t(\hat{\mathbf{r}})\, G_r(\hat{\mathbf{r}})\, \Gamma(\vec{r})$$

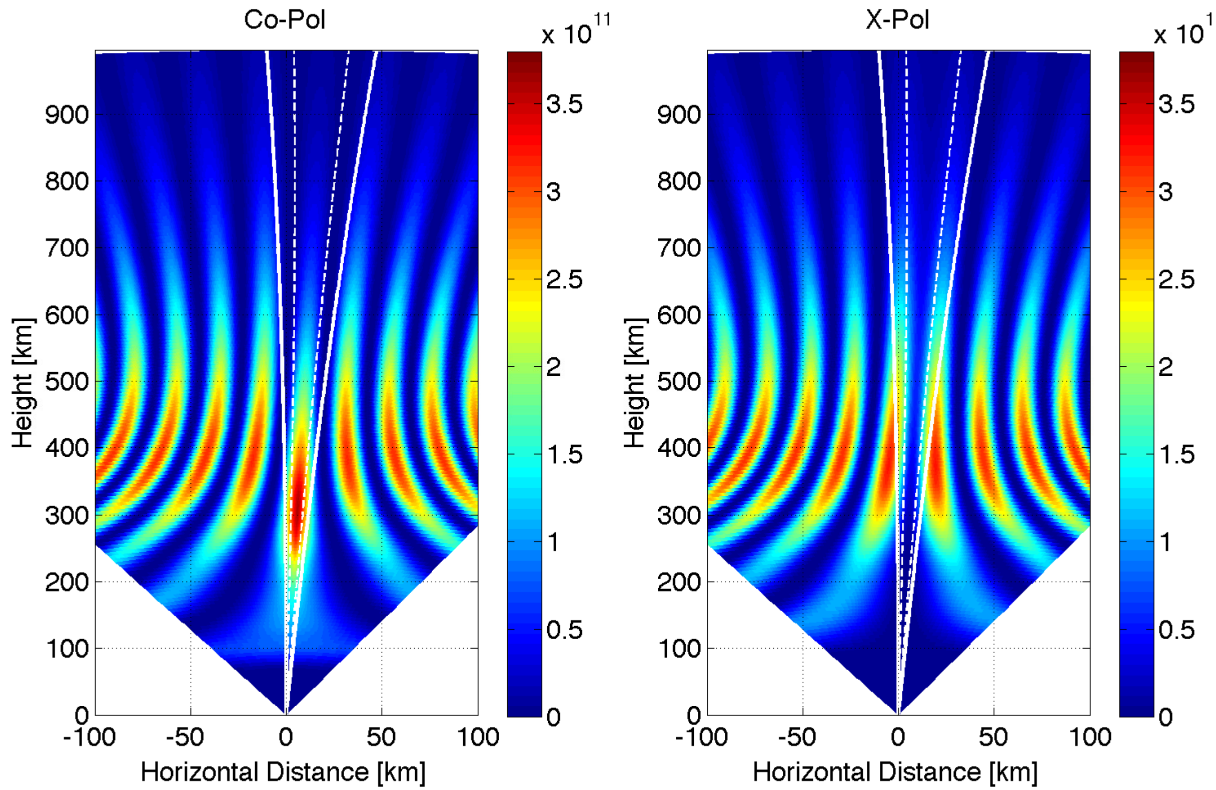
where $\Gamma(\vec{r})$ is a polarization coefficient

$$\Gamma(\vec{r}) = \left| \hat{\mathbf{p}}_r^{\mathsf{T}}\, \bar{\mathbf{\Pi}}(\vec{r})\, \hat{\mathbf{p}}_t \right|^2$$

polarization unit vectors

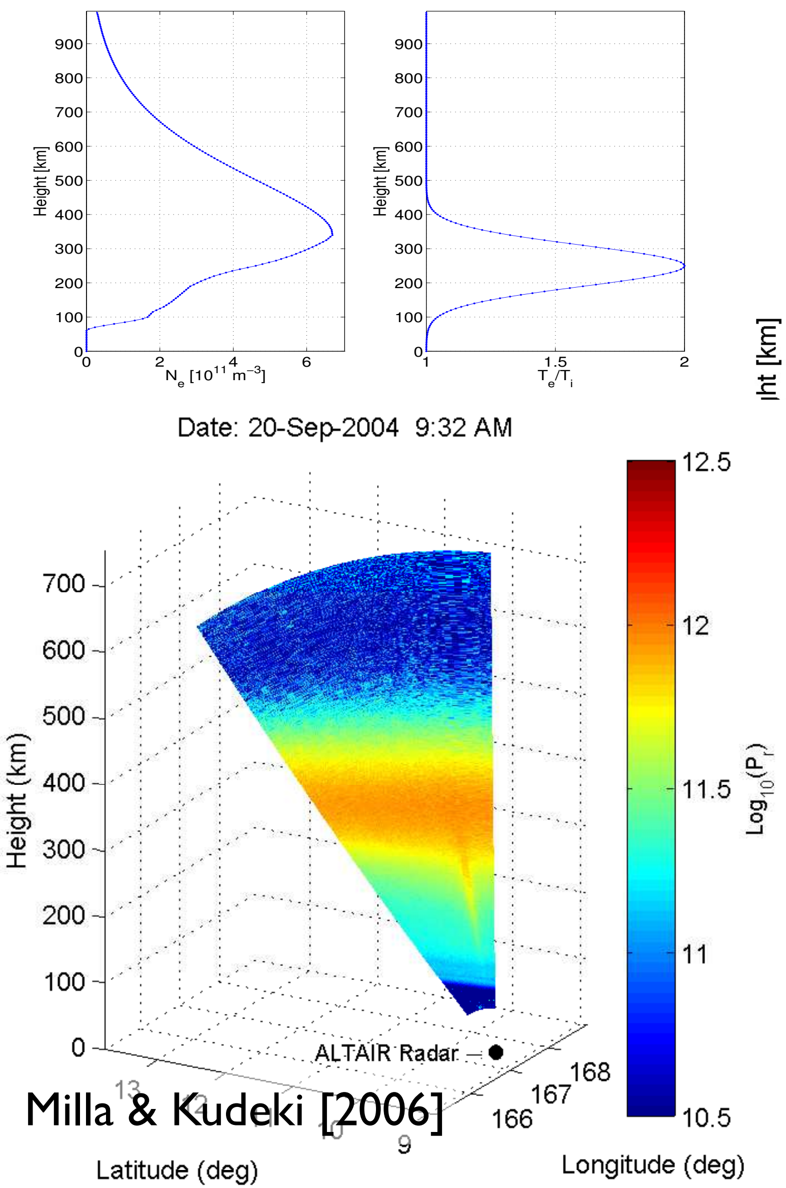
Magneto-ionic propagation model (3)



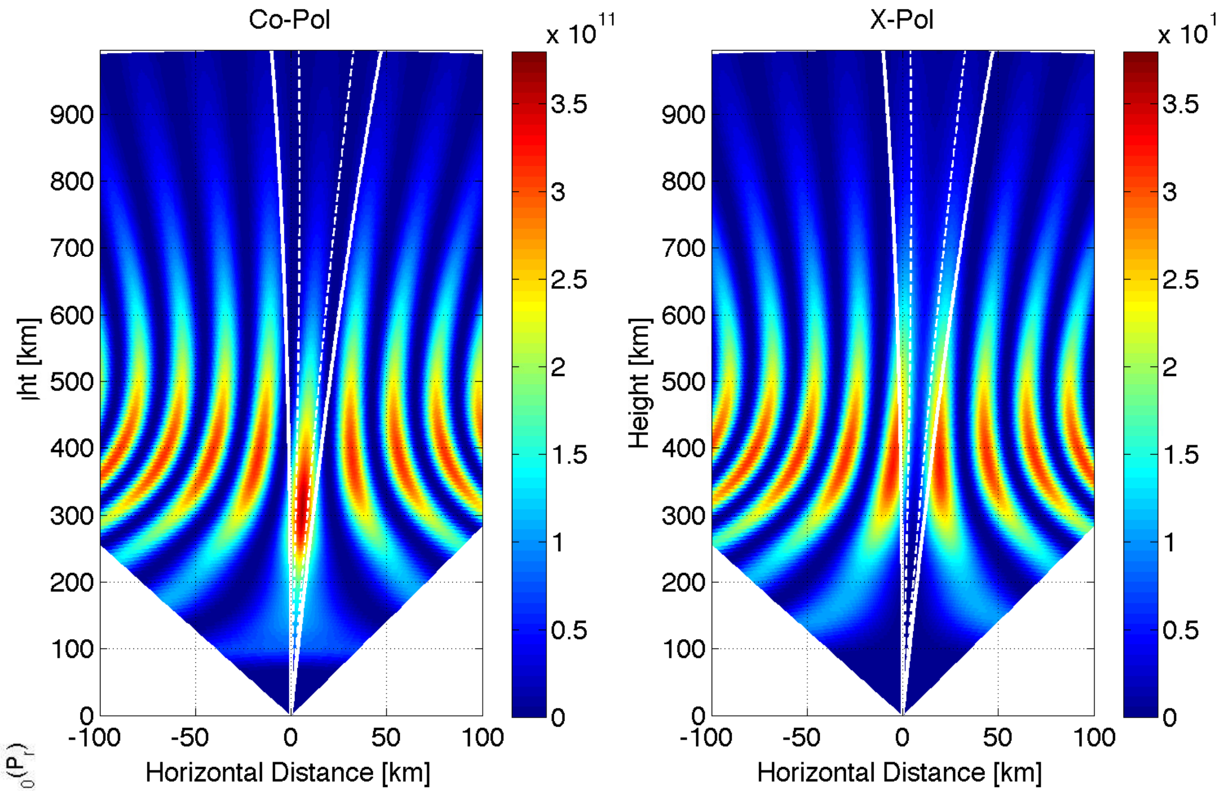


Co-polarized, cross-polarized, and total backscattered power detected by a pair of orthogonal linearly polarized antennas.

Magneto-ionic propagation model (3)

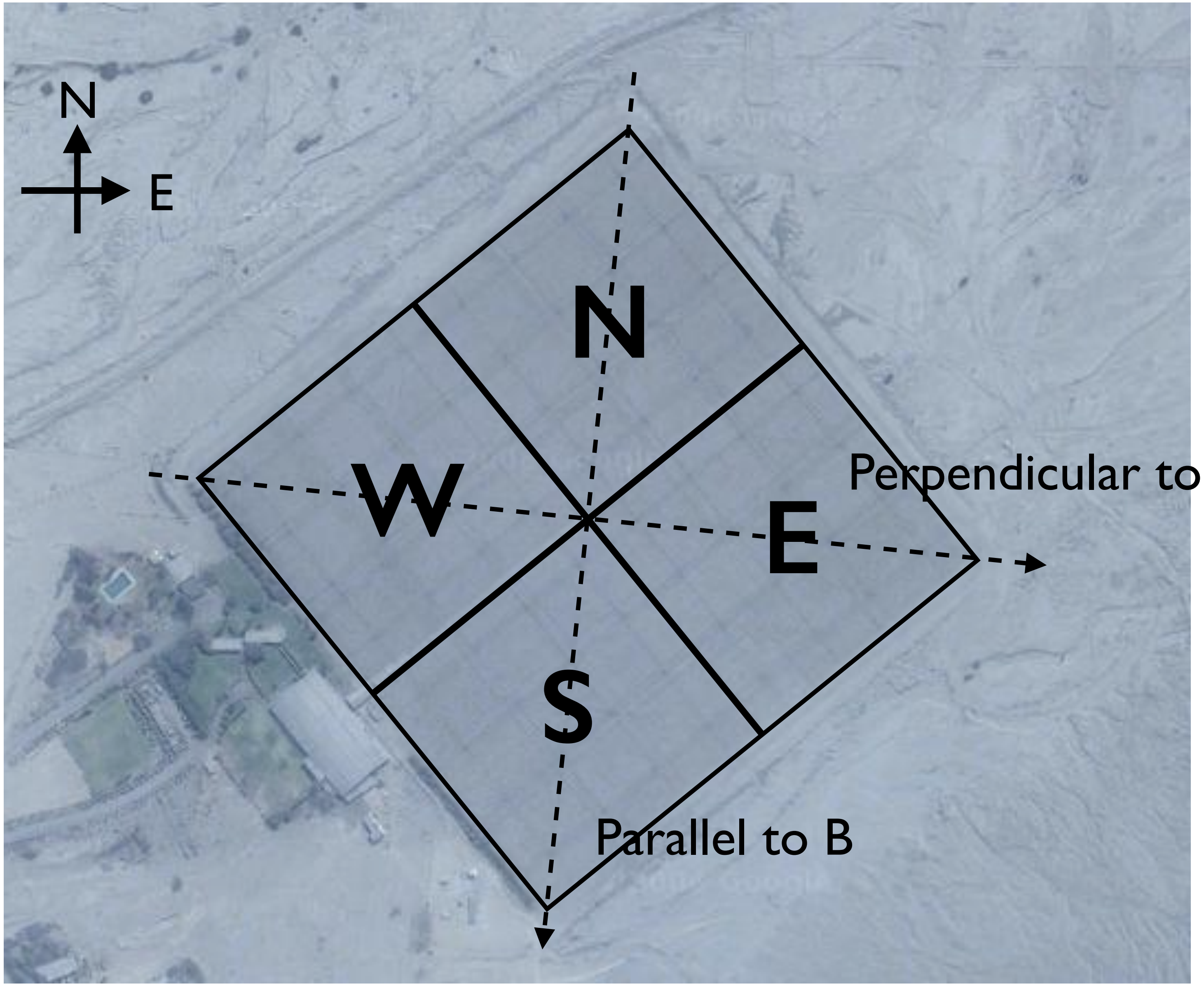


Milla & Kudeki [2006]



Co-polarized, cross-polarized, and total backscattered power detected by a pair of orthogonal linearly polarized antennas.

Application: Differential-phase experiment



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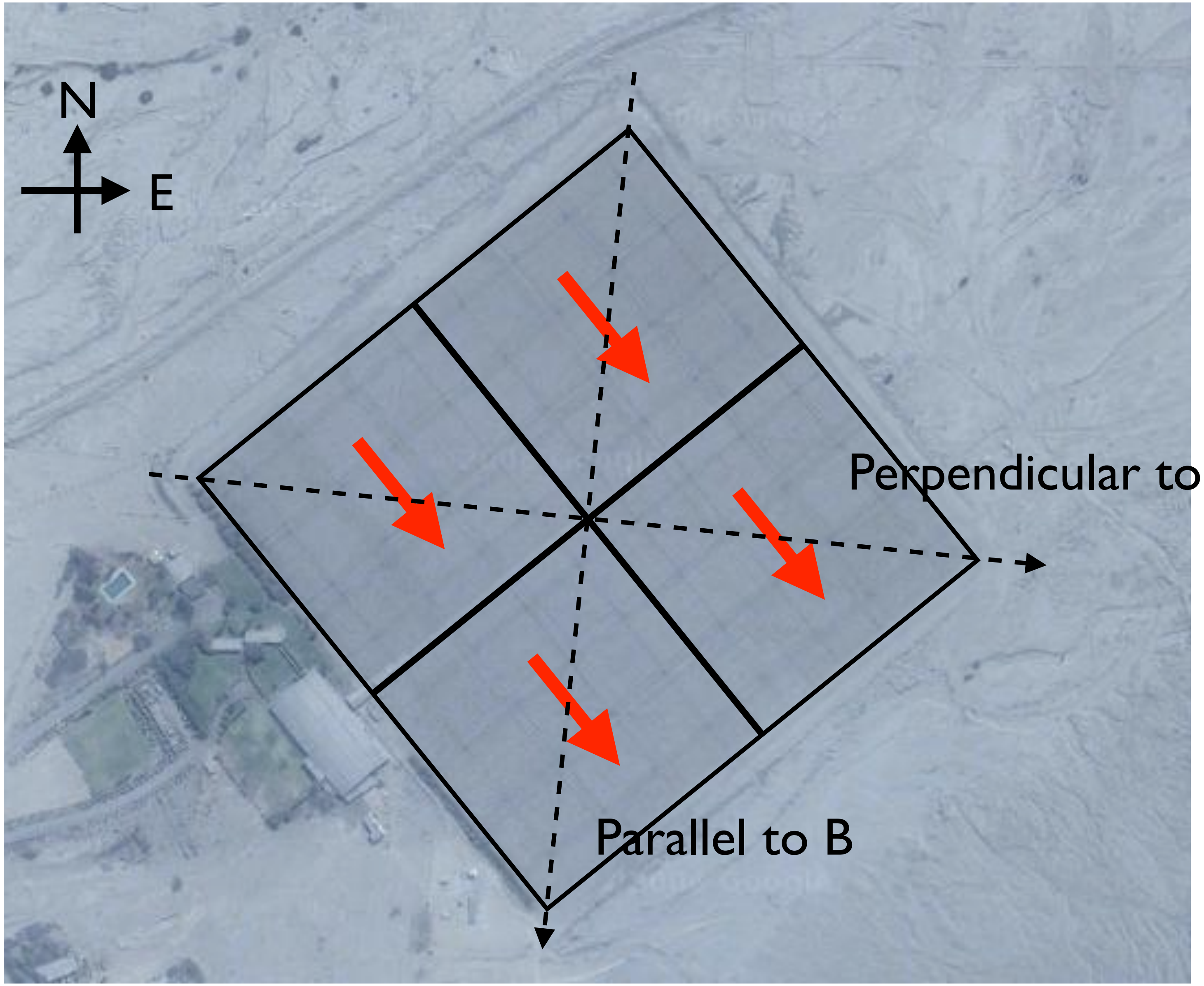

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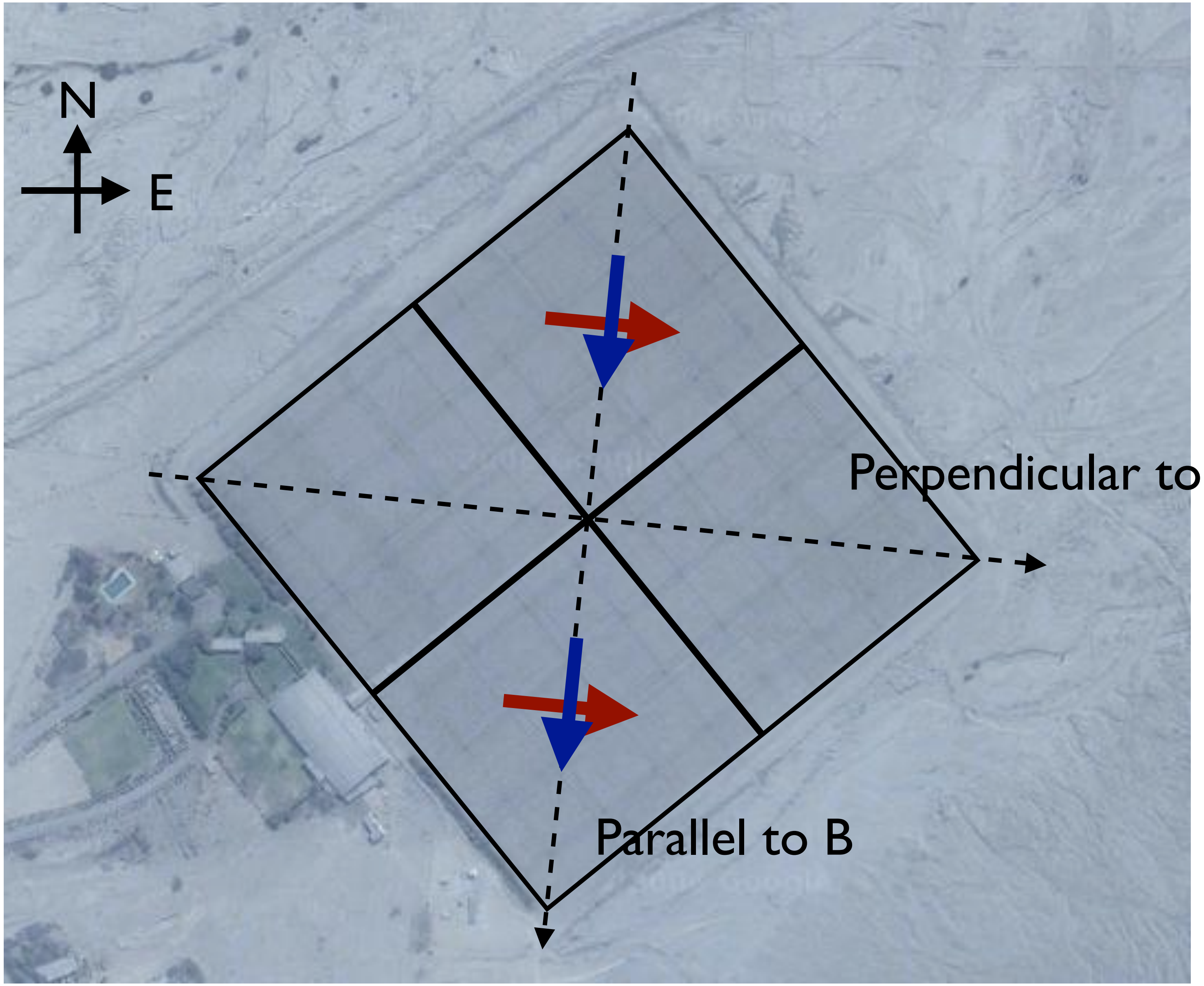
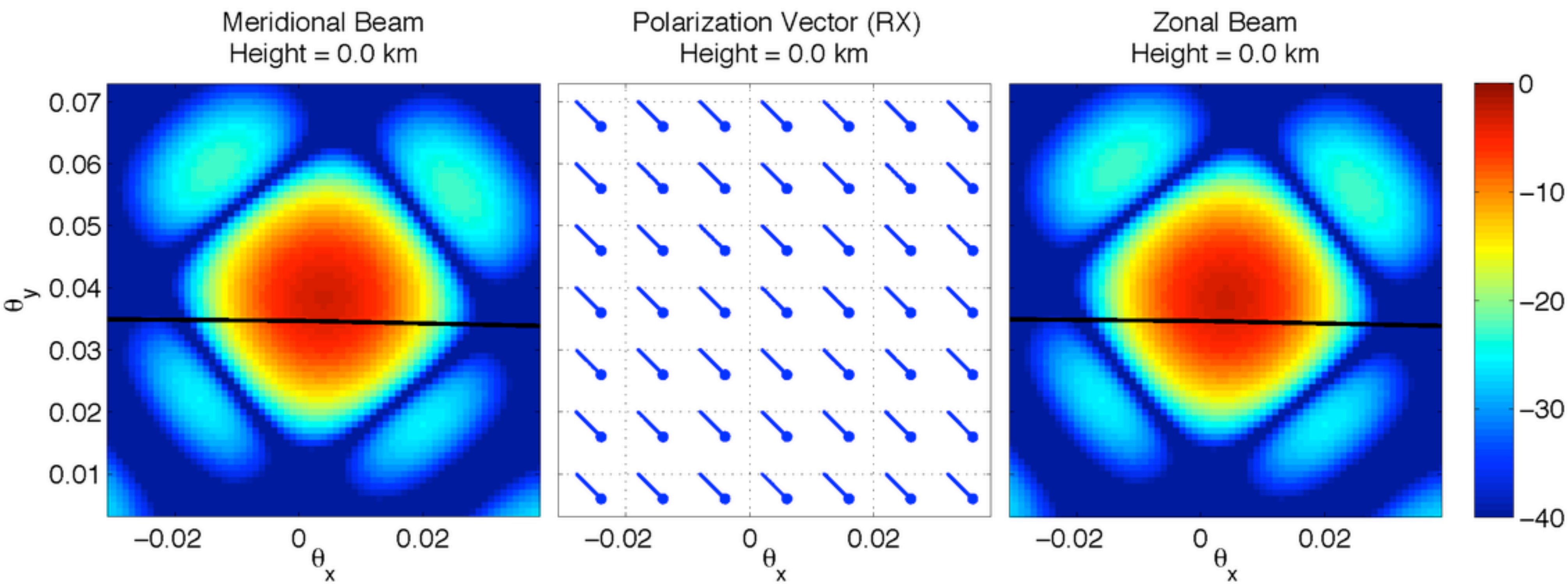
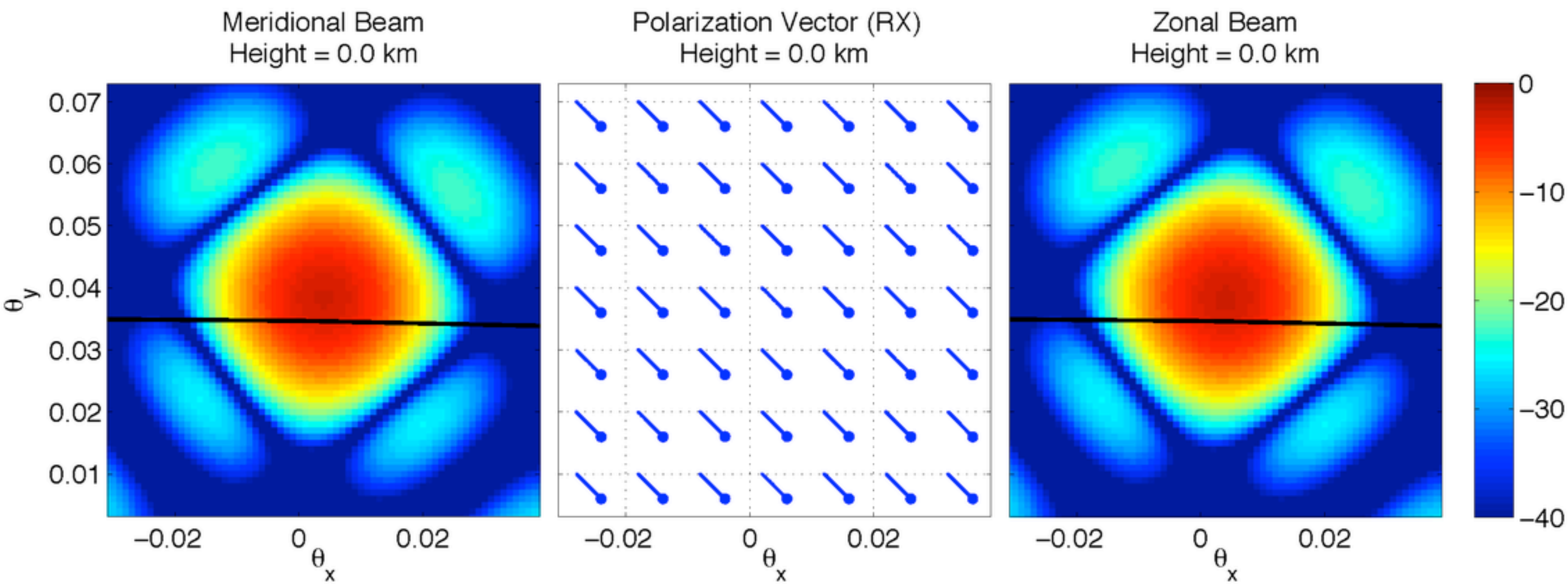


Beam-shape modified by magneto-ionic propagation effects



Simulation

Beam-shape modified by magneto-ionic propagation effects

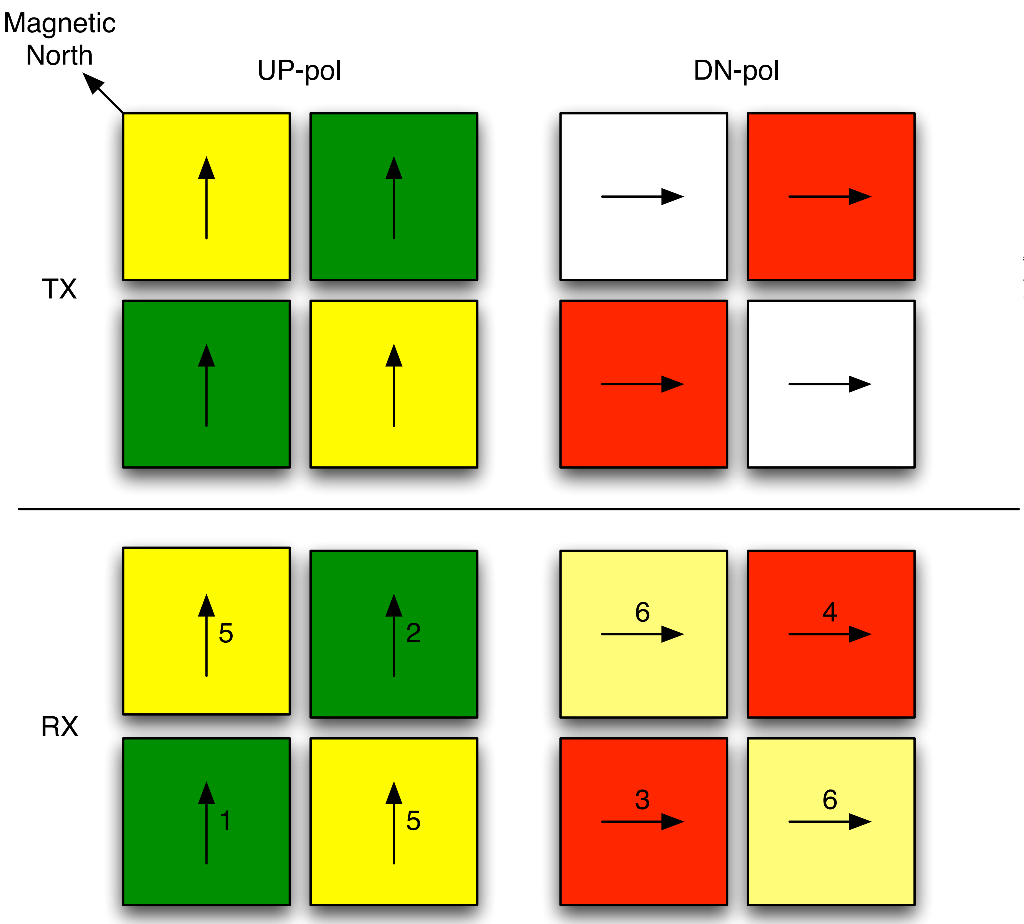


Simulation

Application:

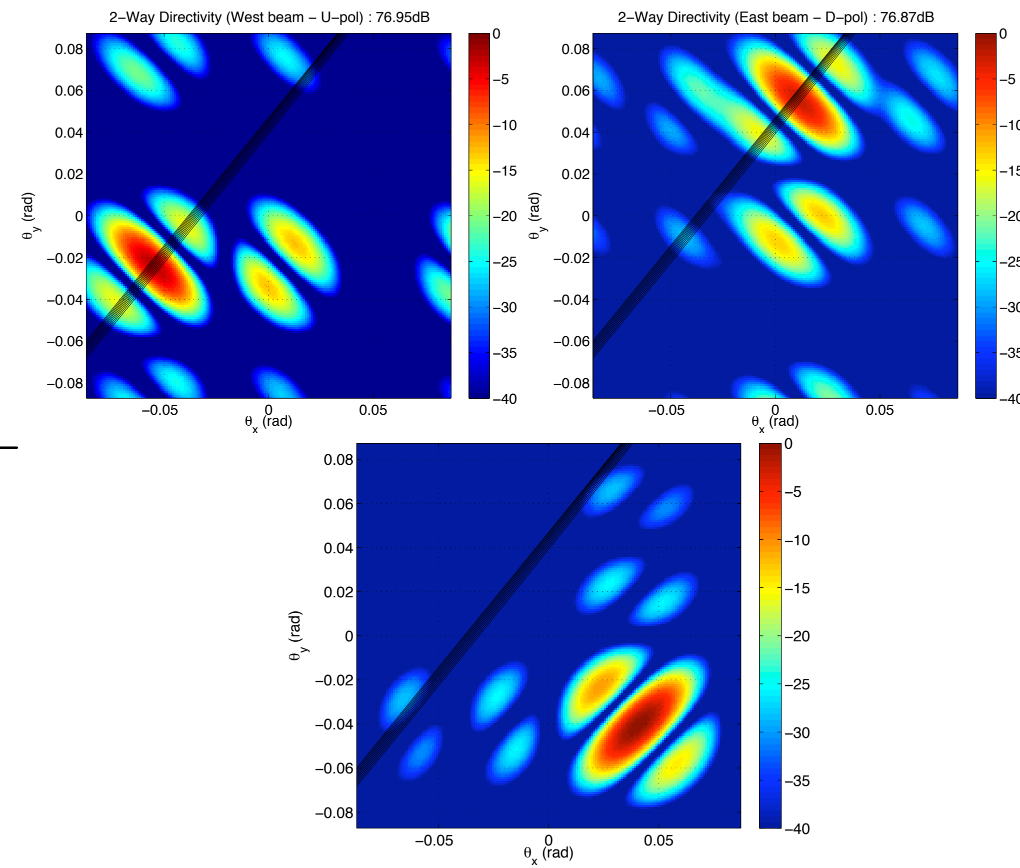
3-Beam radar experiment and estimation of Ne and Te/Ti profiles

3Ba antenna configuration



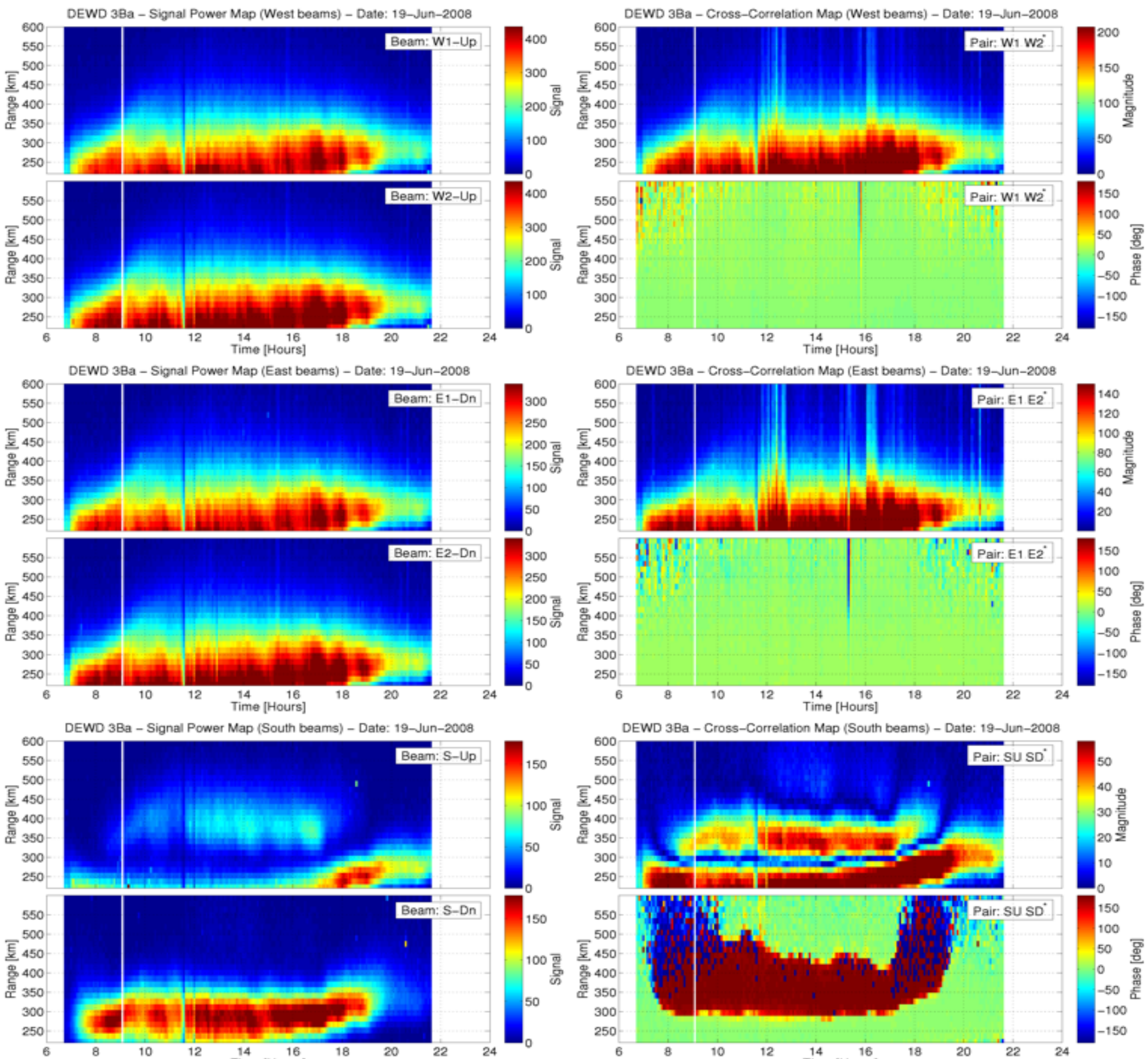
RX1 WU: West 1 (Quarter)
RX2 EU: West 2 (Quarter)
RX5 SU-NU: South (Co-Pol)

RX3 WD: East 1 (Quarter)
RX4 ED: East 2 (Quarter)
RX6 SD-ND: South (X-Pol)



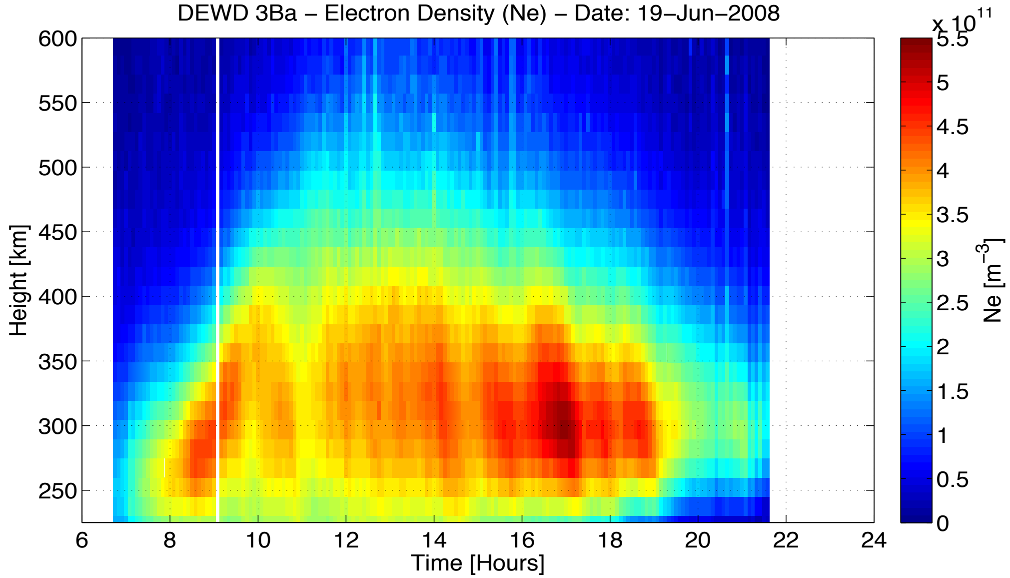
West, east, and south radiation patterns

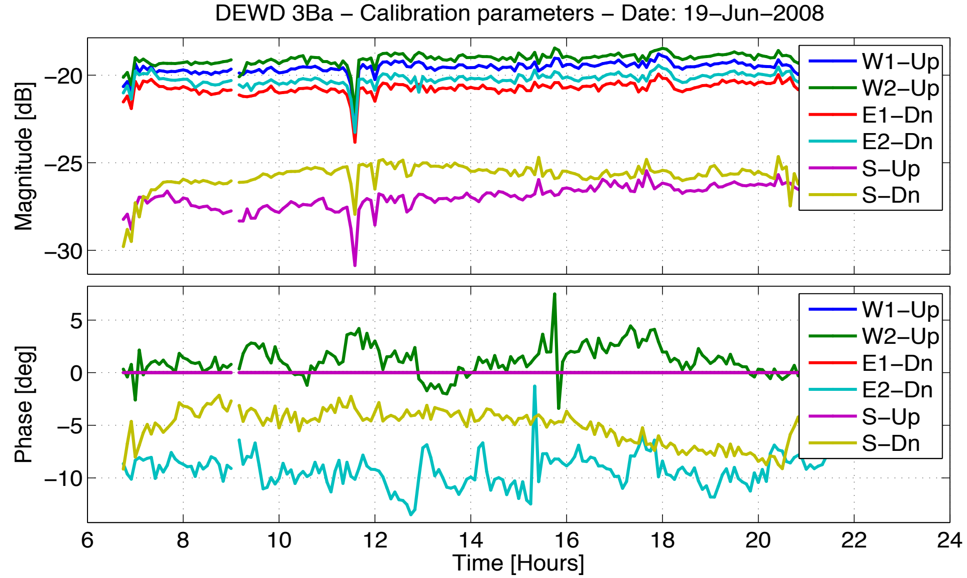
Power and cross-correlation measurements

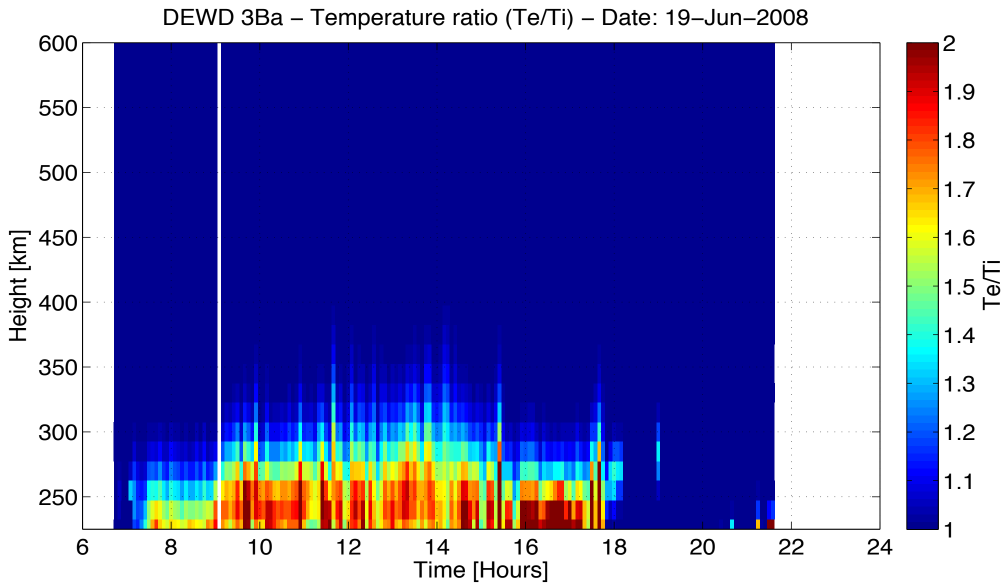


- Range vs. time plots of the signal power and cross-correlation data measured in the 3Ba experiment of June 19, 2008.
- On the left, the power data collected by each of the radar beams are displayed in linear scale.
- On the right, the magnitudes of the cross-correlation data are also plotted in linear scale, while the phase data are plotted in degrees.

Inversion results







Estimated parameters

- Electron densities
- Te/Ti profiles
- Calibration parameters

Conclusions and Future work

- The modeling of the perpendicular-to-B IS spectrum measured by the Jicamarca radar needs to consider:
  - Electron and ion Coulomb collisions effects
  - Magneto-ionic propagation effects
  - Beam-weighting effects
- We have developed the tools to model these effects, but still need to optimize our procedure for routine operational use.
- We also need to study in more detail the sensitivity of our model to plasma temperatures and densities.
- Our model was developed for an O+ plasma, we need to extend our model to H+ and He+ plasmas for radar observations of the topside.
- Spectral fitting for Te estimation should now be possible given the Te/Ti profiles and the development of our collisional ISR spectral model.